HW Solution §7.1 Ed 8. (Last Two Pages for Ed 7)

2. $\int \sqrt{x} \ln x \, dx$.

$$\boxed{\begin{array}{ll} u = \ln x. & dv = \sqrt{x}\, dx \\ du = \frac{1}{x} dx & v = \frac{2}{3} x^{\frac{3}{2}} \end{array}} \quad v = \int \sqrt{x}\, dx = \frac{2}{3} x^{\frac{3}{2}}$$

$= \int u \cdot dv$

$= u \cdot v - \int v \cdot du = \ln x \cdot \frac{2}{3} x^{\frac{3}{2}} - \int \frac{2}{3} x^{\frac{3}{2}} \cdot \frac{1}{x} dx.$

$= \frac{2}{3} \ln x \cdot x^{\frac{3}{2}} - \int \frac{2}{3} \cdot x^{\frac{1}{2}} dx.$

$= \boxed{\frac{2}{3} \ln x \cdot x^{\frac{3}{2}} - \frac{2}{3} \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} + C}$

3. $\int x \cos 5x \, dx$ $\qquad u = x, \quad dv = \cos 5x \cdot dx.$

$\qquad du = dx, \quad v = \int \cos 5x \cdot dx = \frac{1}{5} \sin 5x.$

$= \int u \cdot dv = u \cdot v - \int v \, du$

$= x \cdot \frac{1}{5} \sin 5x - \int \frac{1}{5} \sin 5x \cdot dx.$

$= \frac{1}{5} \cdot x \cdot \sin 5x - \frac{1}{5} \cdot (-\frac{1}{5}) \cos 5x + C$

$= \boxed{\frac{1}{5} x \cdot \sin 5x + \frac{1}{25} \cos 5x + C}$

5. $\int t e^{-3t} dt.$ $\qquad u = t, \quad dv = e^{-3t} dt$

$\qquad du = dt, \quad v = \int e^{-3t} dt = -\frac{1}{3} e^{-3t}$

$= \int u \cdot dv = u \cdot v - \int v \cdot du$

$= t \cdot (-\frac{1}{3}) e^{-3t} - \int (-\frac{1}{3} e^{-3t}) \, dt$

$= -\frac{t}{3} \cdot e^{-3t} + \frac{1}{3} \int e^{-3t} dt$

$= -\frac{t}{3} e^{-3t} + \frac{1}{3} \cdot \boxed{\frac{1}{-3} \cdot e^{-3t}} + C$

$= \boxed{-\frac{t}{3} \cdot e^{-3t} - \frac{1}{9} \cdot e^{-3t} + C}$

☆☆☆ 9. $\int \cos^{-1}x \, dx$. Hint: $(\cos^{-1}x)' = -\frac{1}{\sqrt{1-x^2}}$

$= \cos^{-1}x \cdot x - \int x \cdot \left(-\frac{dx}{\sqrt{1-x^2}}\right)$

$\boxed{u = \cos^{-1}x, \quad dv = dx \\ du = -\frac{1}{\sqrt{1-x^2}}dx, \quad v = x}$

IBP first.

$= \cos^{-1}x \cdot x + \int \frac{x}{\sqrt{1-x^2}} dx$

$= \cos^{-1}x \cdot x + \int \frac{\frac{du}{-2}}{\sqrt{u}}$

Then u-Sub, $u = 1-x^2$, $du = -2x \cdot dx \Rightarrow \frac{du}{-2} = x\,dx$

$= \cos^{-1}x \cdot x - \frac{1}{2} \cdot \underline{\int u^{-\frac{1}{2}} du}$

$= \cos^{-1}x \cdot x - \frac{1}{2} \cdot \underline{2 \cdot u^{\frac{1}{2}}} + C = \boxed{\cos^{-1}x \cdot x - (1-x^2)^{\frac{1}{2}} + C}$

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11. $\int t^4 \cdot \ln t \cdot dt$

$u = \ln t, \quad dv = t^4 dt$

$u = \frac{1}{t} \cdot dt, \quad v = \int t^4 \cdot dt = \frac{1}{5}t^5$

$= \int \ln t \cdot \underline{t^4 dt}$

$= \int u \cdot dv = u \cdot v - \int v \cdot du = \ln t \cdot \frac{1}{5}t^5 - \int \frac{1}{5} \cdot t^5 \cdot \frac{1}{t} \cdot dt$

$= \ln t \cdot \frac{1}{5}t^5 - \int \frac{1}{5} \cdot t^4 dt$

$= \boxed{\frac{1}{5}\ln t \cdot t^5 - \frac{1}{5} \cdot \frac{1}{5} \cdot t^5 + C}$

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15. $\int (\ln x)^2 dx$,

$\boxed{u = (\ln x)^2, \quad dv = dx \\ du = 2\ln x \cdot \frac{1}{x}dx, \quad v = x}$

Hint: IBP twice

$= (\ln x)^2 \cdot x - \int x \cdot 2\ln x \cdot \frac{1}{x} dx$

$= (\ln x)^2 \cdot x - \underline{\int 2\ln x \cdot dx}$ ← IBP again: $u = 2\ln x, \quad dv = dx$; $du = 2\frac{1}{x}dx, \quad v = x$.

$= (\ln x)^2 \cdot x - \left[2\ln x \cdot x - \int x \cdot 2 \cdot \frac{1}{x} dx\right]$

$= (\ln x)^2 \cdot x - \left[2\ln x \cdot x - \int 2 \cdot dx\right]$

$= \boxed{(\ln x)^2 x - (2\ln x \cdot x - 2x) + C}$

28 $\int_0^{2\pi} t^2\cdot \sin 2t \cdot dt$

Method 1: IBP twice.

1st IBP:
$u = t^2, \quad dv = \sin 2t \cdot dt$
$du = 2t\cdot dt, \quad V = \int \sin 2t\, dt = -\frac{1}{2}\cos 2t.$

$= t^2\cdot\left(-\frac{1}{2}\cos 2t\right) - \int \left(-\frac{1}{2}\cos 2t\right) \underline{2t\cdot dt}$

$= t^2\cdot\left(-\frac{1}{2}\cos 2t\right) - \int \underline{2t}\ \underbrace{\left(-\frac{1}{2}\cos 2t\right)dt}$

$= t^2\cdot\left(-\frac{1}{2}\cos 2t\right) - \left[2t\left(-\frac{1}{4}\sin 2t\right) - \int\left(-\frac{1}{4}\sin 2t\right)\cdot 2dt\right]$

$= t^2\cdot\left(-\frac{1}{2}\cos 2t\right) - \left[2t\left(-\frac{1}{4}\sin 2t\right) - 2\times\left(-\frac{1}{4}\right)\cdot\left(-\frac{1}{2}\right)\cdot\cos 2t\right]$

2nd IBP:
$u = 2t, \quad dv = -\frac{1}{2}\cos 2t\cdot dt$
$du = 2\cdot dt, \quad V = \int -\frac{1}{2}\cos 2t$
$= \left(-\frac{1}{2}\right)\cdot\frac{1}{2}\sin 2t$
$= -\frac{1}{4}\cdot\sin 2t$

$= \boxed{t^2\cdot\left(-\frac{1}{2}\cdot\cos 2t\right) - 2t\cdot\left(-\frac{1}{4}\sin 2t\right) + 2\times\left(-\frac{1}{4}\right)\cdot\left(-\frac{1}{2}\right)\cdot\cos 2t \Big|_0^{2\pi}}$

$= -\frac{1}{2}t^2\cdot\cos 2t + \frac{t}{2}\cdot\sin 2t + \frac{1}{4}\cdot\cos 2t \Big|_0^{2\pi}$

$\cos 4\pi = 1$
$\cos 0 = 1$

$= -\frac{1}{2}(2\pi)^2\cdot\cos 4\pi + \frac{2\pi}{2}\sin 4\pi + \frac{1}{4}\cdot\cos 4\pi - \left(0 + 0 + \frac{1}{4}\cos 0\right)$

$= -2\pi^2 + 0 + \frac{1}{4} - \frac{1}{4} = \boxed{-2\pi^2}$

Method 2: Table

$u = t^2, \quad dv = \sin 2t\, dt \Rightarrow V = \frac{1}{-2}\cos 2t.$

derivative ↓	$2t$	$\frac{1}{-2}\cdot\frac{1}{2}\sin 2t$
	2	$\frac{1}{-2}\cdot\frac{1}{2}\cdot\left(\frac{1}{-2}\right)\cos 2t$

anti-Derivative (Integration) ↓

$\int t^2\cdot\sin 2t\, dt = \boxed{t^2\cdot\left(\frac{1}{-2}\cos 2t\right)} - \boxed{2t\times\frac{1}{-2}\cdot\frac{1}{2}\sin 2t} + \boxed{2\times\frac{1}{-2}\cdot\frac{1}{2}\cdot\frac{1}{-2}\cdot\cos 2t}$

$+C$

39. $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\, d\theta$ $\boxed{☆☆☆☆}$

Step 1: u-sub: $u=\theta^2$, $du=2\theta\cdot d\theta$

$$\frac{du}{2\theta}=d\theta.$$

$$\int_{\theta=\sqrt{\frac{\pi}{2}}}^{\theta=\sqrt{\pi}} \xrightarrow{u=\theta^2} \int_{u=\frac{\pi}{2}}^{u=\pi}$$

$$\int_{\theta=\sqrt{\frac{\pi}{2}}}^{\theta=\sqrt{\pi}} \theta^3\cdot\cos(\theta^2)\cdot d\theta = \int_{u=\pi/2}^{u=\pi} \theta^3\cos u\cdot\frac{du}{2\theta}$$

$$= \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\theta^2\cdot\cos u\cdot du$$

$$= \int_{\frac{\pi}{2}}^{\pi} \underbrace{\frac{1}{2}\cdot u}_{t}\cdot\underbrace{\cos u\, du}_{dv}$$

Step 2: IBP

$t=\frac{1}{2}\cdot u$, $dv=\cos u\cdot du$

$dt=\frac{1}{2}du$, $v=\int\cos u\, du=\sin u$

$$\int_{\frac{\pi}{2}}^{\pi}\frac{1}{2}u\cdot\cos u\cdot du = \int t\cdot dv$$

$$= t\cdot v-\int v\cdot dt$$

$$= \cancel{\theta}\frac{1}{2}\cdot u\cdot\sin u-\int\sin u\cdot\frac{1}{2}du$$

$$= \frac{1}{2}u\cdot\sin u-\frac{1}{2}\cdot(-\cos u)\Big|_{\frac{\pi}{2}}^{\pi}$$

$\sin\pi=0$, $\cos\pi=-1$

$\sin\frac{\pi}{2}=1$, $\cos\frac{\pi}{2}=0$

$$= \frac{1}{2}\cdot\pi\cdot\sin\pi+\frac{1}{2}\cos\pi-\left(\frac{1}{2}\cdot\frac{\pi}{2}\sin\frac{\pi}{2}+\frac{1}{2}\cdot\cos\frac{\pi}{2}\right)$$

$$= 0-\frac{1}{2}-\left(\frac{\pi}{4}\cdot 1+0\right)$$

$$= \boxed{-\frac{1}{2}-\frac{\pi}{4}}$$

41. $\int x\cdot\ln(1+x)\,dx$ - ☆☆☆

u-sub first. $u=1+x$. $du=dx$, $x=u-1$

$=\int \underbrace{(u-1)}\cdot\underbrace{\ln u}\cdot du$ Then IBP

$t=\ln u$, $dv=(u-1)du$

$dt=\frac{1}{u}du$, $v=\int(u-1)du$

$=\frac{1}{2}u^2-u$.

$=\int \underbrace{\ln u}\cdot\underbrace{(u-1)du}$

$=\int t\,dv = t\cdot v-\int v\cdot dt$

$=\ln u\cdot(\frac{1}{2}u^2-u)-\int(\frac{1}{2}u^2-u)\cdot\frac{1}{u}du$

$=\ln u\cdot(\frac{1}{2}u^2-u)-\int(\frac{1}{2}u-1)du$

$=\ln u\cdot(\frac{1}{2}u^2-u)-(\frac{1}{2}\cdot\frac{1}{2}u^2-u)+C$

$u\mapsto x+1$

$=\boxed{\ln(1+x)\left[\frac{1}{2}(1+x)^2-(1+x)\right]-\left(\frac{1}{4}(1+x)^2\right)+(1+x)+C}$

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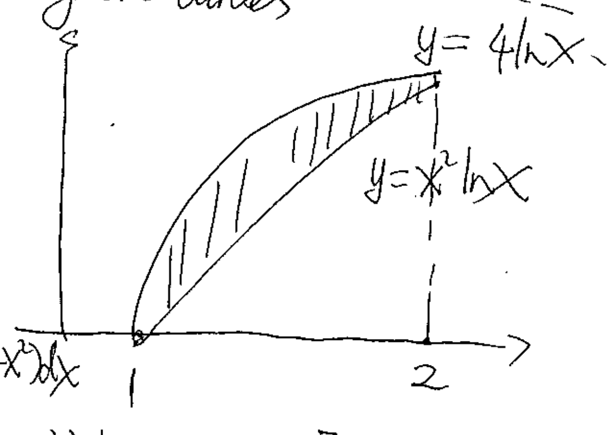
5. Find the area of the region bdd by the given curves ☆☆☆☆

$4\ln x = x^2\cdot\ln x$.

$\Rightarrow x=1$ or $4=x^2$, $x=2$



Area $=\int_1^2 4\ln x - x^2\ln x\,dx$

$=\int_1^2 \underbrace{(4-x^2)}_{dv}\underbrace{\ln x}_{u}\,dx$

IBP: $u=\ln x$, $dv=(4-x^2)dx$

$du=\frac{1}{x}dx$, $v=\int(4-x^2)dx=4x-\frac{1}{3}x^3$

$=\ln x\cdot(4x-\frac{1}{3}x^3)-\int(4x-\frac{1}{3}x^3)\cdot\frac{1}{x}\cdot dx$

$=\ln x\cdot(4x-\frac{1}{3}x^3)-\int(4-\frac{1}{3}x^2)dx=\boxed{\ln x\cdot(4x-\frac{1}{3}x^3)-(4x-\frac{1}{9}x^3)\Big|_1^2}$

$=\frac{16}{3}\ln 2-\frac{29}{9}$

70 If $f(0)=g(0)=0$, prove.

☆☆☆☆☆. $\int_0^a \underbrace{f(x)}_{u}\cdot \underbrace{g''(x)\cdot dx} = f(a)\cdot g'(a) - f'(a)g(a) + \int_0^a f''(x)\cdot g(x)\,dx$

Solution: IBP twice.

1st: $u = f(x)$, $dv = g''(x)\,dx$

$du = f'(x)\,dx$, $v = \int g''(x)\,dx = g'(x)$

$$\int f(x)\cdot g''(x)\,dx = \int u\,dv = u\cdot v - \int v\cdot du$$
$$= f(x)\cdot g'(x) - \int \underbrace{g'(x)}_{dv}\cdot \underbrace{f'(x)}_{u}\cdot dx$$

2nd IBP: $u = f'(x)$, $dv = g'(x)\,dx$.

$du = f''(x)\cdot dx$, $v = \int g'(x)\,dx = g(x)$

$$\int g'(x)\cdot f'(x)\,dx = \int u\cdot dv = u\cdot v - \int v\cdot du$$
$$= \underbrace{f'(x)\cdot g(x) - \int g(x)\cdot f''(x)\,dx}$$

Plug back

$$\int f(x)\cdot g''(x)\,dx = f(x)\cdot g'(x) - \int g'(x)\cdot f'(x)\,dx$$
$$= f(x)\cdot g'(x) - \left[f'(x)\cdot g(x) - \int g(x) f''(x)\,dx\right]$$
$$= f(x)\cdot g'(x) - f'(x)\cdot g(x) + \int g(x)\cdot f''(x)\,dx$$

$$\int_0^a f(x)\cdot g''(x)\,dx = f(x)\cdot g'(x) - f'(x)\cdot g(x)\Big|_0^a + \int_0^a g(x)\cdot f''(x)\,dx$$
$$= f(a)\cdot g'(a) - f'(a)\cdot g(a) - 0 + \int_0^a g(x)\cdot f''(x)\,dx.$$

H/W. Solution. §7.1 Ed7.

1. $\int x^2 \cdot \ln x\, dx$; $\boxed{\begin{array}{ll} u=\ln x, & dv=x^2 dx \\ du=\frac{1}{x}dx, & v=\int x^2 dx=\frac{1}{3}x^3 \end{array}}$

$$=\int u\cdot dv = uv-\int v\,du = \ln x\cdot\tfrac{1}{3}x^3 - \int \tfrac{1}{3}x^3\cdot\tfrac{1}{x}\,dx$$

$$=\ln x\cdot\tfrac{1}{3}x^3 - \int\tfrac{1}{3}\cdot x^2\cdot dx$$

$$=\boxed{\ln x\cdot\tfrac{1}{3}\cdot x^3 - \tfrac{1}{3}\cdot\tfrac{1}{3}x^3 + C}$$

#3. the same as #3 in Ed 8.

#5. - - - as #5 in Ed 8

10 $\int \sin^{-1}x \cdot dx$. Hint: $(\sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$

IBP first, then u-sub

$\boxed{\begin{array}{ll} u=\sin^{-1}x, & dv=dx \\ du=\frac{1}{\sqrt{1-x^2}}dx, & v=x \end{array}}$ IBP

$$=u\cdot v-\int v\cdot du$$

$$=\sin^{-1}x\cdot x - \underbrace{\int x\cdot\frac{1}{\sqrt{1-x^2}}dx}_{\text{u-sub, then}}$$

u-sub, then, $\boxed{\begin{array}{l} u=1-x^2, \quad du=-2\cdot x\,dx \\ \frac{1}{-2}\cdot du = dx \end{array}}$ u-sub

$$=\sin^{-1}x\cdot x - \int\frac{\frac{1}{-2}\cdot du}{\sqrt{u}}$$

$$=\sin^{-1}x\cdot x + \tfrac{1}{2}\cdot\int u^{-\frac{1}{2}}du = \sin^{-1}x\cdot x + \tfrac{1}{2}\cdot 2\cdot\sqrt{u} + C$$

$$=\boxed{\sin^{-1}x\cdot x + \sqrt{1-x^2} + C}$$

12. $\int p^5 \ln p\, dp$ $\boxed{\begin{array}{ll} u=\ln p, & dv=p^5 dp \\ du=\frac{1}{p}dp, & v=\frac{1}{6}p^6 \end{array}}$

$$=\int u\cdot v-\int v\cdot du$$

$$=\ln p\cdot\tfrac{1}{6}p^6 - \int\tfrac{1}{6}p^6\cdot\tfrac{1}{p}\,dp$$

$$=\ln p\cdot\tfrac{1}{6}p^6 - \tfrac{1}{6}\int p^5\,dp = \boxed{\ln p\cdot\tfrac{1}{6}p^6 - \tfrac{1}{6}\cdot\tfrac{1}{6}\cdot p^6 + C}$$

15. The same as # 15 in Ed 8. See Ed 8 Solution

28. - - - - - as # 28 in Ed 8.

39. - - same as # 39 in Ed 8. (☆☆☆☆)

41. - - - - - - # 41 in Ed 8. ☆☆☆

57. - - - - - # 57 in Ed 8 ☆☆☆☆

68 See # 70 in Ed 8 ☆☆☆☆