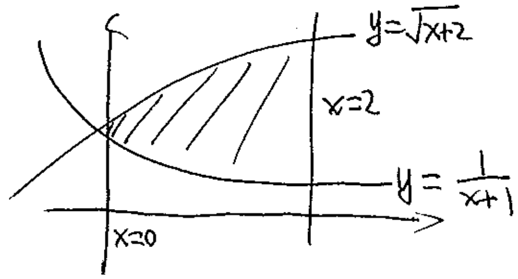
§ ~~6.1~~ 5.1

1-4 Find the area of the shaded region.

2.



[Hint: Upper curve – lower curve]

sln: $A = \int_0^2 [\sqrt{x+2} - \frac{1}{x+1}]\, dx$

$= \int_0^2 \sqrt{x+2}\, dx - \int_0^2 \frac{1}{x+1}\, dx$

$\int_0^2 \sqrt{x+2}\, dx \overset{u=x+2}{\underset{du=dx}{=\!=\!=}} \int_{u(0)}^{u(2)} \sqrt{u} \cdot du = \int_2^4 u^{\frac{1}{2}}\, du$

$= \frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1} \Big|_2^4$

$= \frac{2}{3} u^{\frac{3}{2}} \Big|_2^4 = \frac{2}{3} \cdot 4^{\frac{3}{2}} - \frac{2}{3} \cdot 2^{\frac{3}{2}}$

$\int_0^2 \frac{1}{x+1}\, dx \overset{u=x+1}{\underset{du=dx}{=\!=\!=}} \int_{u(0)}^{u(2)} \frac{1}{u}\, du = \int_1^3 \frac{1}{u}\, du = \ln u \Big|_1^3 = \ln 3 - \ln 1 = \ln 3$

$A = \frac{2}{3} \cdot 4^{\frac{3}{2}} - \frac{2}{3} \cdot 2^{\frac{3}{2}} - \ln 3$

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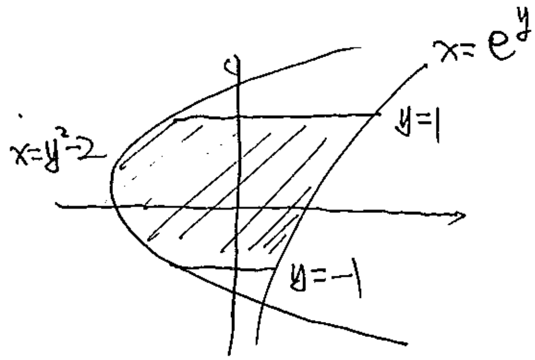
3.



[Hint: Right curve – left curve]

sln: $A = \int_{-1}^{1} [e^y - (y^2 - 2)]\, dy$

$= \int_{-1}^{1} (e^y - y^2 + 2)\, dy$

$= e^y - \frac{1}{3} y^3 + 2y \Big|_{-1}^{1}$

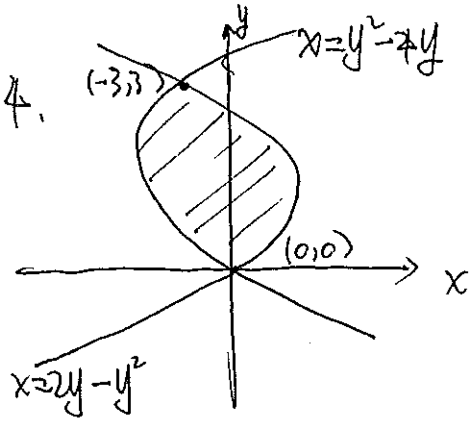
$= (e^1 - \frac{1}{3} \cdot 1^3 + 2 \cdot 1) - [e^{-1} - \frac{1}{3} \cdot (-1)^3 + 2 \cdot (-1)]$

$= e - \frac{1}{3} + 2 - e^{-1} - \frac{1}{3} + 2$

$= e - e^{-1} + \frac{10}{3}$

#

4.



Hint: find the intersection points of $\begin{cases} x=y^2-4y \\ x=2y-y^2 \end{cases}$

one is $(-3,3)$, one is $(0,0)$

sln: $A=\int_0^3 [2y-y^2]-(y^2-4y)\,dy$

$= \int_0^3 (6y-2y^2)\,dy$

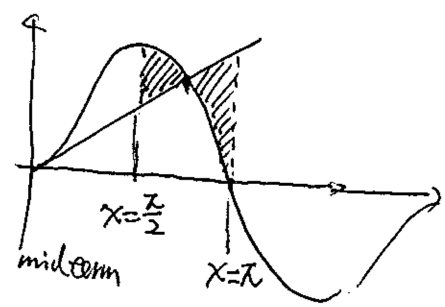
$=\left(6\cdot\frac{1}{2}y^2-2\cdot\frac{1}{3}y^3\right)\Big|_0^3$

$=(3\cdot 3^2-\frac{2}{3}\cdot 3^3)-(0-0)=27-18=9$

5-12. Sketch the region enclosed by the curves and find the area.

6. $y=\sin x$, $y=x$, $x=\frac{\pi}{2}$, $x=\pi$.

sln:



Remark: this problem will not appear in quiz or midterm since it needs to solve the solution $\sin x = x$ (unsolvable)

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8. $y=x^2-2x$, $y=x+4$.

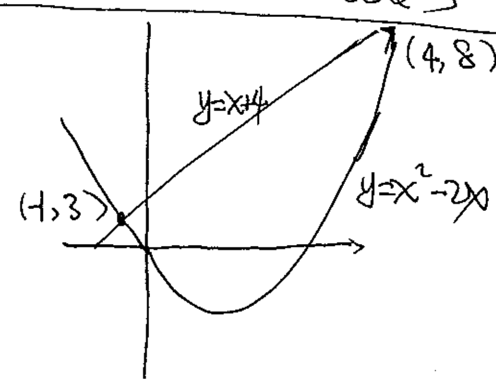
sln: solve intersection points.

$\begin{cases} y=x^2-2x \\ y=x+4 \end{cases} \Rightarrow x^2-2x=x+4$

$x^2-3x-4=0$

$x=-1,\ 4$ $\quad(-1,3)$

$y=3,\ 8$ $\quad(4,8)$



upper curve: $y=x+4$, lower curve: $y=x^2-2x$.

$$A = \int_{-1}^{4} \left[(x+4) - (x^2 - 2x)\right] dx$$

$$= \int_{-1}^{4} (x + 4 - x^2 + 2x)\, dx$$

$$= \int_{-1}^{4} (3x + 4 - x^2)\, dx$$

$$= \left(3 \cdot \frac{1}{2}x^2 + 4x - \frac{1}{3}x^3\right)\Big|_{-1}^{4}$$

$$= \left(\frac{3}{2} \cdot 4^2 + 4 \cdot 4 - \frac{1}{3} \cdot 4^3\right) - \left[\frac{3}{2} \cdot (-1)^2 + 4 \cdot (-1) - \frac{1}{3} \cdot (-1)^3\right]$$

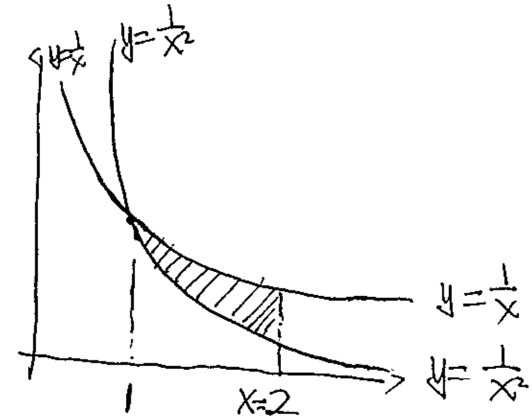
$$= 24 + 16 - \frac{64}{3} - \frac{3}{2} + 4 - \frac{1}{3} = 44 - \frac{65}{3} - \frac{3}{2}$$

9. $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $x = 2$

sln: intersection points

$$\begin{cases} y = \frac{1}{x} \\ y = \frac{1}{x^2} \end{cases} \Rightarrow \frac{1}{x} = \frac{1}{x^2} \Rightarrow x^2 = x$$

$$\Rightarrow x = 1$$



$$A = \int_{1}^{2} \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

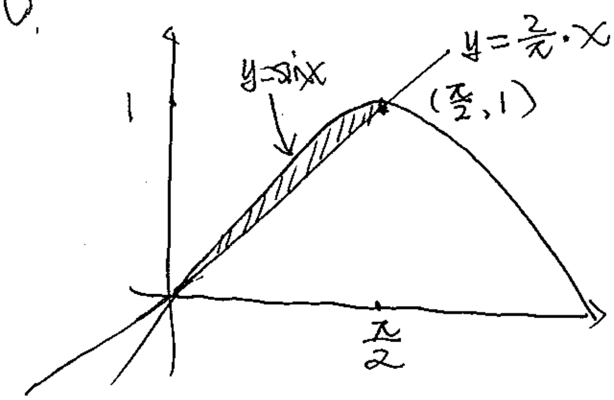
Hint: $\frac{1}{x^2} = x^{-2}$

$$= \left(\ln x - \frac{1}{-2+1} \cdot x^{-2+1}\right)\Big|_{1}^{2}$$

$$= (\ln x + x^{-1})\Big|_{1}^{2} = \left(\ln 2 + \frac{1}{2}\right) - (\ln 1 + 1^{-1}) = \ln 2 + \frac{1}{2} - 1$$

$$= \ln 2 - \frac{1}{2}$$

10. $y = \sin x$, $y = \frac{2x}{\pi}$, $x \geq 0$.



sln: $A = \int_{0}^{\frac{\pi}{2}} \left(\sin x - \frac{2}{\pi} \cdot x\right) dx$

$$= \left(-\cos x - \frac{2}{\pi} \cdot \frac{1}{2}x^2\right)\Big|_{0}^{\frac{\pi}{2}}$$

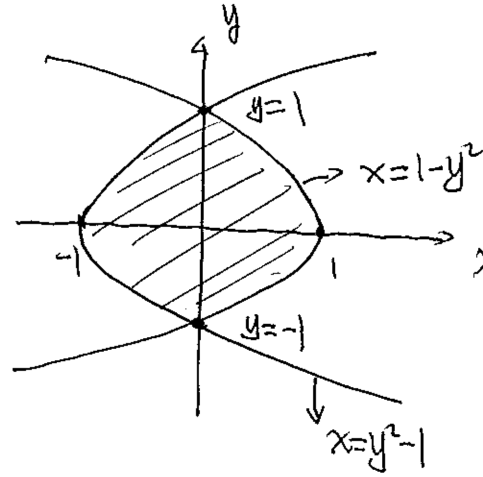
$$= \left(-\cos x - \frac{1}{\pi} x^2\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \left[-\cos\frac{\pi}{2} - \frac{1}{\pi}\cdot\left(\frac{\pi}{2}\right)^2\right] - \left[-\cos 0 - \frac{1}{\pi}\cdot 0^2\right]$$

$$= 0 - \frac{1}{\pi}\cdot\frac{\pi^2}{4} + 1 - 0 = 1 - \frac{\pi}{4}$$

※.

11. $x = 1 - y^2$, $x = y^2 - 1$



sln: intersection points

$$\begin{cases} x = 1 - y^2 \\ x = y^2 - 1 \end{cases} \Rightarrow 1 - y^2 = y^2 - 1$$

$$\Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1$$

$$\Rightarrow y = -1, 1.$$

right curve: $x = 1 - y^2$, left curve: $x = y^2 - 1$

$$A = \int_{-1}^{1} \left[(1 - y^2) - (y^2 - 1)\right] dy = \int_{-1}^{1} (2 - 2y^2)\, dy$$

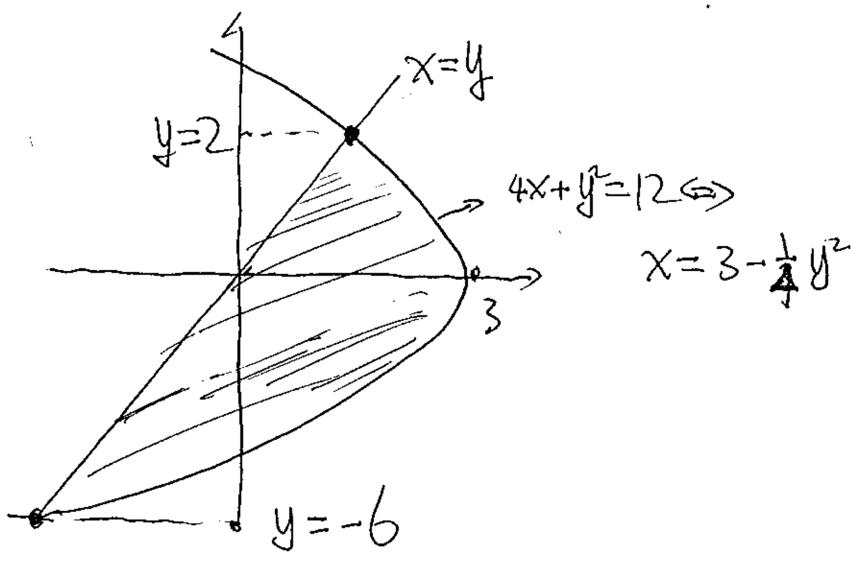
$$= \left(2y - 2\cdot\frac{1}{3}y^3\right)\Big|_{-1}^{1} = 2\cdot 1 - \frac{2}{3}\cdot 1^3 - \left(2\cdot(-1) - \frac{2}{3}\cdot(-1)^3\right)$$

$$= 2 - \frac{2}{3} + 2 - \frac{2}{3}$$

$$= \frac{8}{3}$$

※.

12. $4x + y^2 = 12$, $x = y$



sln: intersection points

$$\begin{cases} x = 3 - \frac{1}{4}y^2 \\ x = y \end{cases} \Rightarrow 3 - \frac{1}{4}y^2 = y$$

$$\Rightarrow \frac{1}{4}y^2 + y - 3 = 0$$

$$\Rightarrow y^2 + 4y - 12 = 0$$

$$y = -6,\ y = 2$$

right curve: $x=3-\frac{1}{4}y^2$, left curve: $x=y$

$$A=\int_{-6}^{2}\left[3-\tfrac{1}{4}y^2\right]-y\ dy$$

$$=\int_{-6}^{2}3-\tfrac{1}{4}y^2-y\ dy$$

$$=\left(3y-\tfrac{1}{4}\cdot\tfrac{1}{3}y^3-\tfrac{1}{2}\cdot y^2\right)\Big|_{-6}^{2}$$

$$=\left(3y-\tfrac{1}{12}y^3-\tfrac{1}{2}y^2\right)\Big|_{-6}^{2}$$

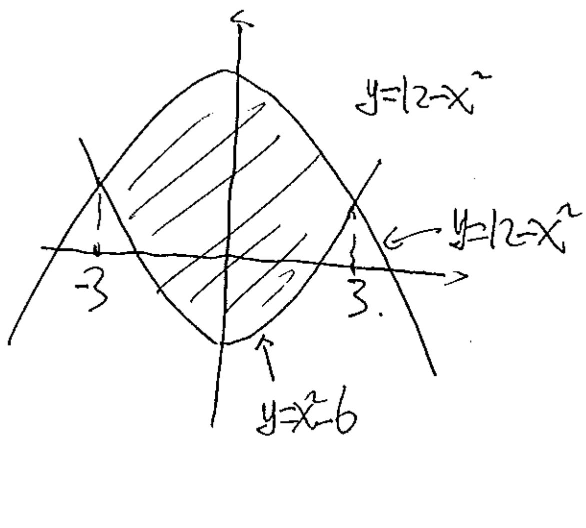
$$=3\cdot2-\tfrac{1}{12}\cdot2^3-\tfrac{1}{2}\cdot2^2-\left[3\cdot(-6)-\tfrac{1}{12}\cdot(-6)^3-\tfrac{1}{2}\cdot(-6)^2\right]$$

$$=6-\tfrac{8}{12}-2+\underline{18-18}+18=22-\tfrac{2}{3}$$

※

13-28 sketch the region enclosed by the given curves and find its area.

13. $y=12-x^2$, $y=x^2-6$.



sln: intersection points.

$$\begin{cases}y=12-x^2\\ y=x^2-6\end{cases}\Rightarrow 12-x^2=x^2-6$$

$$\Rightarrow 2x^2=18$$

$$x^2=9$$

$$x=-3,\ 3$$

upper curve $y=12-x^2$, lower curve: $y=x^2-6$.

$$A=\int_{-3}^{3}\left[(12-x^2)-(x^2-6)\right]dx$$

$$=\int_{-3}^{3}(18-2x^2)\,dx$$

$$=\left(18x-\tfrac{2}{3}x^3\right)\Big|_{-3}^{3}$$

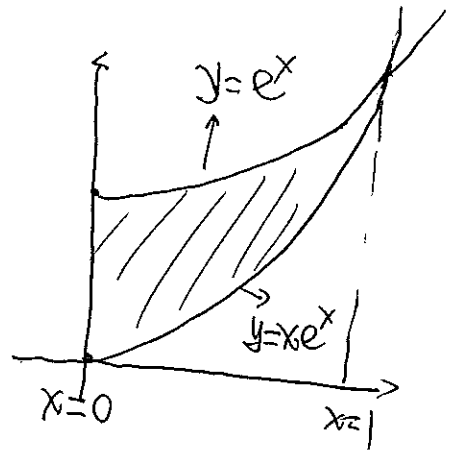
$$=\left(18\cdot3-\tfrac{2}{3}\cdot3^3\right)-\left(18\cdot(-3)-\tfrac{2}{3}\cdot(-3)^3\right)$$

$$=54-18+54-18$$

$$=72$$

※

15. $y=e^x$, $y=x\cdot e^x$, $x=0$



s/n:

intersection point.

$e^x = x\cdot e^x \Rightarrow x=1$

upper : $y=e^x$, lower : $y=xe^x$

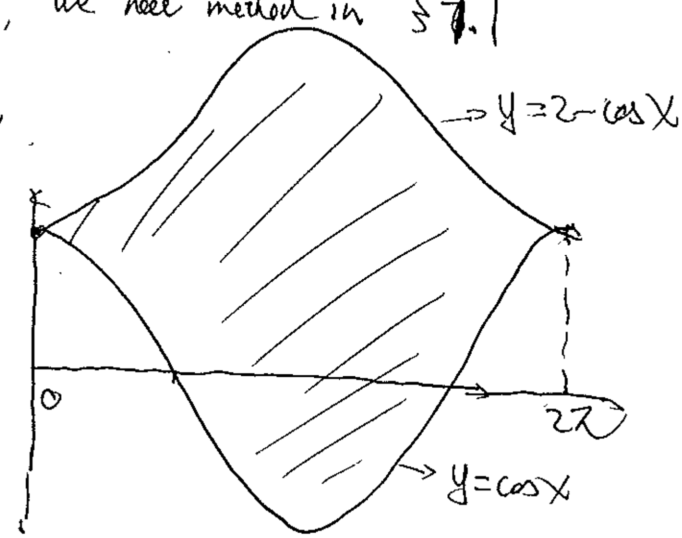
$$A=\int_0^1 [e^x - x\cdot e^x]\, dx$$

※

Remark this problem will not appear in this quiz or the midterm

~~the~~ in order to evaluate this problem, we need method in §7.1

16. $y=\cos x$, $y=2-\cos x$, $0\le x\le 2\pi$



s/n:

intersection points:

$\cos x = 2-\cos x$

$\Rightarrow 2\cos x = 2 \Rightarrow \cos x = 1$

$\Rightarrow x=0,\ 2\pi.$

upper curve: $y=2-\cos x$, lower curve : $y=\cos x$

$$A=\int_0^{2\pi}[(2-\cos x)-\cos x]\,dx$$

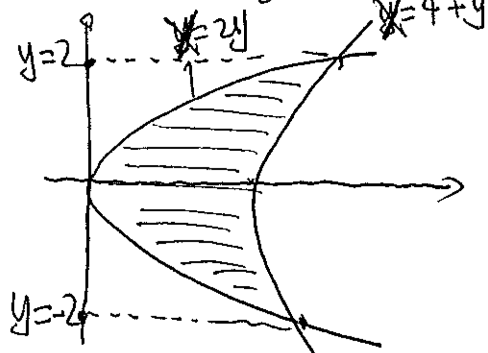
$$=\int_0^{2\pi}(2-2\cos x)\,dx$$

$$=(2x-2\sin x)\Big|_0^{2\pi}$$

$$=(2\cdot 2\pi - 2\cdot\sin 2\pi)-(2\cdot 0 - 2\cdot \sin 0)$$

$$=4\pi - 0 - 0 + 0 = 4\pi$$ ※

17. $x=2y^2$, $x=4+y^2$



sln:

intersection points:

$\begin{cases} x=2y^2 \\ x=4+y^2 \end{cases} \Rightarrow 2y^2=4+y^2,$

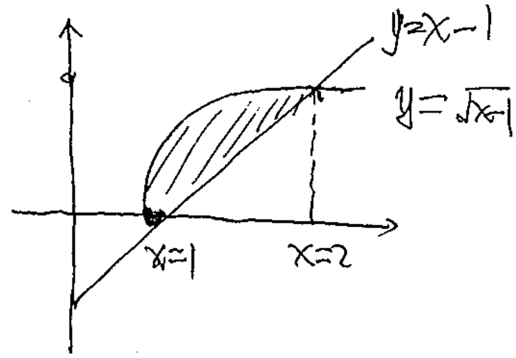
$\Rightarrow y^2=4 \Rightarrow y=-2, 2$. right curve: $x=4+y^2$. left curve $x=2y^2$.

$$A=\int_{-2}^{2}(4+y^2)-2y^2\,dy$$

$$=\int_{-2}^{2}(4-y^2)\,dy=(4y-\tfrac{1}{3}y^3)\Big|_{3}^{3} = 4\cdot 3-\tfrac{1}{3}\cdot 3^3-(4\cdot(-3)-\tfrac{1}{3}(-3)^3)$$

$$=12-9+12-9=6$$ ※

18. $y=\sqrt{x-1}$, $x-y=1$



sln:

intersection points:

$\begin{cases} y=\sqrt{x-1} \\ x-y=1 \Rightarrow y=x-1 \end{cases} \Rightarrow \sqrt{x-1}=x-1$

$\Rightarrow x-1=(x-1)^2$

$x-1=x^2-2x+1$, $x^2-3x+2=0$, $x=1$, or 2

upper curve: $y=\sqrt{x-1}$, lower curve: $y=x-1$

$$A=\int_1^2\sqrt{x-1}-(x-1)\,dx \;\overset{u=x-1}{\underset{du=dx}{=}}\; \int_{u(1)}^{u(2)}(\sqrt{u}-u)\,du$$

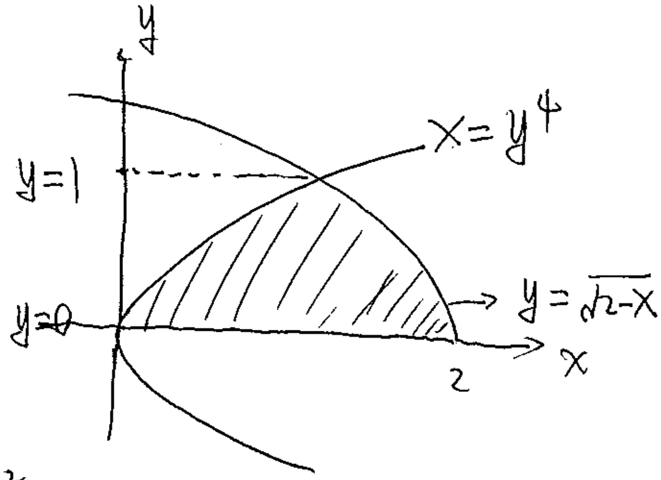
$$=\int_0^1(u^{\frac{1}{2}}-u)\,du$$

$$=\left(\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1}-\frac{1}{2}u^2\right)\Big|_0^1$$

$$=\left(\frac{2}{3}u^{\frac{3}{2}}-\frac{1}{2}u^2\right)\Big|_0^1$$

$$=\left(\frac{2}{3}\cdot 1^{\frac{3}{2}}-\frac{1}{2}\cdot 1^2\right)-(0-0)=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$$ ※

20. $x = y^4$, $y = \sqrt{2-x}$, $y = 0$



s/n: intersection points:

$$\begin{cases} x = y^4 \\ y = \sqrt{2-x} \Rightarrow y^2 = 2-x \Rightarrow x = 2-y^2 \end{cases}$$

$\Rightarrow y^4 = 2 - y^2$

$\Rightarrow (y^2)^2 + y^2 - 2 = 0 \Rightarrow y^2 = 1 \Rightarrow y = 1$, (-1 is not needed since $y \geq 0$)

or $y^2 = -2$ (meaningless)

right $y = \sqrt{2-x}$, left $x = y^4$

$\Leftrightarrow x = 2 - y^2$

$$A = \int_0^1 [(2-y^2) - y^4]\,dy = \int_0^1 (2 - y^2 - y^4)\,dy$$

$$= \left(2y - \tfrac{1}{3}y^3 - \tfrac{1}{5}y^5\right)\Big|_0^1$$

$$= \left(2 - \tfrac{1}{3} - \tfrac{1}{5}\right) - 0 = \frac{22}{15}$$

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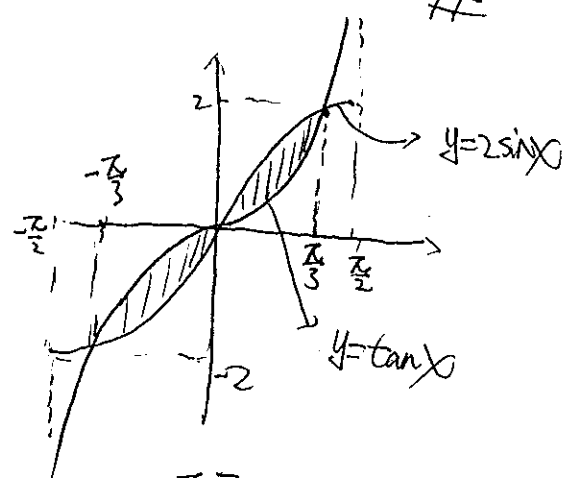
21. $y = \tan x$, $y = 2\sin x$. $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$



s/n: recall $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $2\sin\frac{\pi}{3} = \sqrt{3}$,

$\tan\frac{\pi}{3} = \sqrt{3}$

which means the intersection points are $\pm\frac{\pi}{3}$.

upper curve: $y = 2\sin x$, lower curve $y = \tan x$ on $[0, \frac{\pi}{3}]$, and switch on $[-\frac{\pi}{3}, 0]$

$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} |2\sin x - \tan x|\,dx$$

$$= 2\int_0^{\frac{\pi}{3}} (2\sin x - \tan x)\,dx = 4\int_0^{\frac{\pi}{3}} \sin x - 2\int_0^{\frac{\pi}{3}} \tan x\,dx.$$

$$\int_0^{\frac{\pi}{3}} \sin x\,dx = -\cos x \Big|_0^{\frac{\pi}{3}} = -\cos\frac{\pi}{3} - (-\cos 0) = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{3}} \tan x\,dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\,dx$$

$$\xlongequal[du = -\sin x\,dx]{u = \cos x} \int_{\cos 0}^{\cos\frac{\pi}{3}} \frac{1}{u} \cdot (-du)$$

$$= -\int_1^{\frac{1}{2}} \frac{1}{u}\,du = -\ln u \Big|_1^{\frac{1}{2}}$$

$$= -\ln\frac{1}{2} - (-\ln 1)$$

$$= -\ln 2^{-1} + 0$$

$$= (-1)\cdot(-1)\cdot\ln 2$$

$$= \ln 2$$

Hint: $\ln x^y = y \ln x$

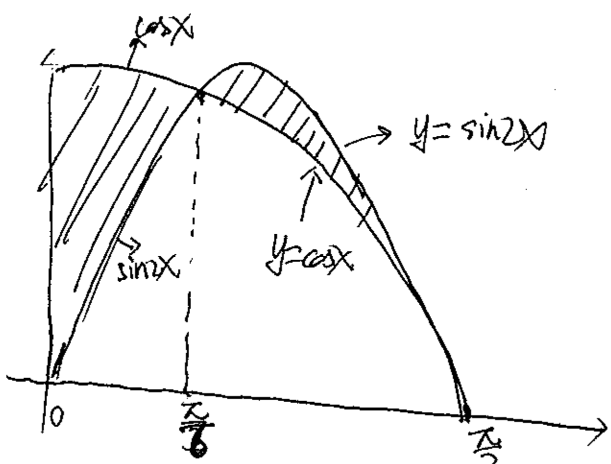
$$A = 4\int_0^{\frac{\pi}{3}} \sin x\,dx - 2\int_0^{\frac{\pi}{3}} \tan x\,dx$$

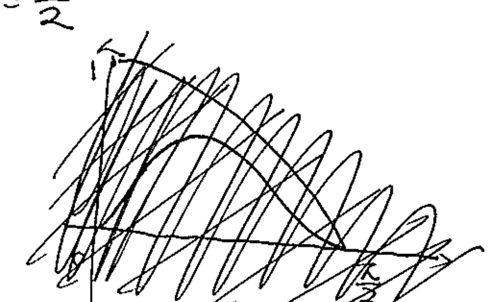
$$= 4\cdot\frac{1}{2} - 2\cdot\ln 2 = 2 - 2\ln 2$$

※

23. $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \frac{\pi}{2}$

S/n:





intersection: $\cos x = \sin 2x$ (Hint: $\sin 2x = 2\sin x \cdot \cos x$)

$\Leftrightarrow \cos x = 2\sin x \cdot \cos x \Leftrightarrow \cos x(1 - 2\sin x) = 0$

$\Leftrightarrow 1 = 2\sin x$ or $\cos x = 0$

$\Leftrightarrow \sin x = \frac{1}{2}$ or $\cos x = 0$, $\Leftrightarrow x = \frac{\pi}{6}$ or $x = \frac{\pi}{2}$.

on $[0; \frac{\pi}{6}]$ upper curve $y=\cos x$, lower curve: $y=\sin 2x$

on $[\frac{\pi}{6}, \frac{\pi}{2}]$ upper $y=\sin 2x$, lower $y=\cos x$

$$A=\int_0^{\frac{\pi}{2}} |\cos x-\sin 2x|\,dx$$

$$=\int_0^{\frac{\pi}{6}} (\cos x-\sin 2x)\,dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x-\cos x)\,dx$$

$$=\int_0^{\frac{\pi}{6}} \cos x\,dx-\int_0^{\frac{\pi}{6}} \sin 2x\,dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\,dx-\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx$$

$$\int_0^{\frac{\pi}{6}} \cos x\,dx=\sin x\Big|_0^{\frac{\pi}{6}}=\sin\frac{\pi}{6}-\sin 0=\frac{1}{2}-0=\frac{1}{2}$$

$$\int_0^{\frac{\pi}{6}} \sin 2x\,dx \;\underset{du=2dx}{\overset{u=2x}{=\!=\!=}}\; \int_0^{\frac{\pi}{3}} \sin u\cdot\frac{du}{2}=\frac{1}{2}\int_0^{\frac{\pi}{3}} \sin u\cdot du$$

$$=\frac{1}{2}(-\cos u)\Big|_0^{\frac{\pi}{3}}$$

$$=\frac{1}{2}(-\cos\frac{\pi}{3})-\left(\frac{1}{2}(-\cos 0)\right)$$

$$=\frac{1}{2}\cdot(-\frac{1}{2})+\frac{1}{2}=-\frac{1}{4}+\frac{1}{2}=\frac{1}{4}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\,dx \;\underset{du=2dx}{\overset{u=2x}{=\!=\!=}}\; \int_{\frac{\pi}{3}}^{\pi} \sin u\,\frac{du}{2}=\frac{1}{2}(-\cos u)\Big|_{\frac{\pi}{3}}^{\pi}=\frac{1}{2}(-\cos\pi)-\left(\frac{1}{2}(-\cos\frac{\pi}{3})\right)$$

$$=\frac{1}{2}(-1)(-1)+\frac{1}{2}\cdot\frac{1}{2}$$

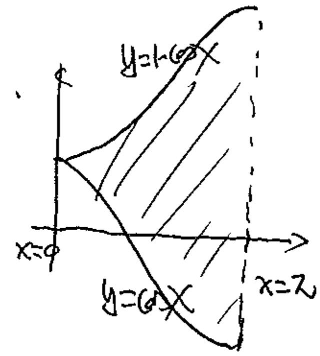
$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx=\sin x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\sin\frac{\pi}{2}-\sin\frac{\pi}{6}=1-\frac{1}{2}=\frac{1}{2}$$

$$A=\frac{1}{2}-\frac{1}{4}+\frac{3}{4}-\frac{1}{2}=\frac{1}{2}$$

※

24. $y=\cos x$, $y=1-\cos x$, $0\le x\le \pi$.



s/n: upper : $y=1-\cos x$

lower : $y=\cos x$

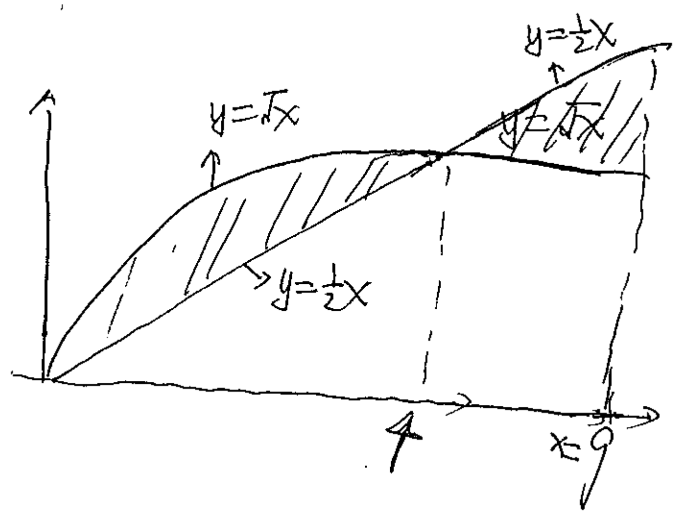
$$A=\int_0^{\pi}(1-\cos x)-\cos x\ dx$$

$$=\int_0^{\pi}1-2\cos x\ dx$$

$$=(x-2\cdot\sin x)\Big|_0^{\pi}=\pi-2\cdot\sin\pi-(0-2\sin 0)=\pi$$

#

25. $y=\sqrt{x}$, $y=\frac{1}{2}x$, $x=9$.



s/n: intersection points :

$x=0$

$\begin{cases} y=\sqrt{x} \\ y=\frac{1}{2}x \end{cases} \Rightarrow \sqrt{x}=\frac{1}{2}x \Rightarrow x=\frac{1}{4}x^2$

$\Rightarrow x=0$ or $x=4$.

	upper	lower
$[0,4]$	$y=\sqrt{x}$	$y=\frac{1}{2}x$
$[4,9]$	$y=\frac{1}{2}x$,	$y=\sqrt{x}$.

$$A=\int_0^9\left|\sqrt{x}-\tfrac{1}{2}x\right|dx=\int_0^4(\sqrt{x}-\tfrac{1}{2}x)\,dx+\int_4^9(\tfrac{1}{2}x-\sqrt{x})\,dx$$

$$=\left(\tfrac{2}{3}x^{\frac{3}{2}}-\tfrac{1}{4}x^2\right)\Big|_0^4+\left(\tfrac{1}{4}x^2-\tfrac{2}{3}\cdot x^{\frac{3}{2}}\right)\Big|_4^9$$

$$=\left(\tfrac{2}{3}\cdot4^{\frac{3}{2}}-\tfrac{1}{4}\cdot4^2\right)-0+\left(\tfrac{1}{4}\cdot9^2-\tfrac{2}{3}\cdot9^{\frac{3}{2}}\right)-\left(\tfrac{1}{4}\cdot4^2-\tfrac{2}{3}\cdot4^{\frac{3}{2}}\right)$$

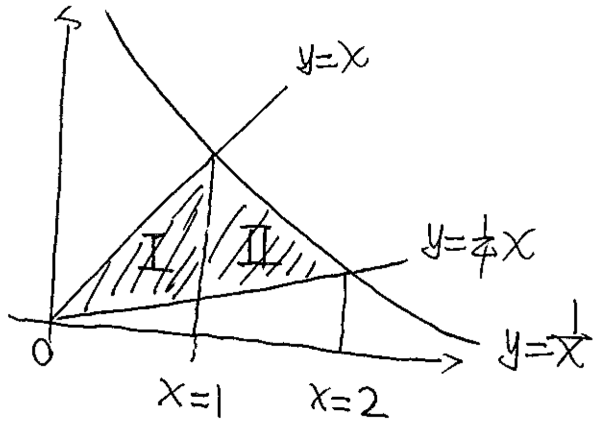
$$=\frac{59}{12}$$

#

27. $y=\frac{1}{x}$, $y=x$, $y=\frac{1}{4}x$, $x>0$

sln: $y=x$ and $y=\frac{1}{x}$ intersect at $x=1$

$y=\frac{1}{4}x$ and $y=\frac{1}{x}$ intersect at $x=2$



$A=$ region I + region II

$$= \int_0^1 (x-\tfrac{1}{4}x)\,dx + \int_1^2 (\tfrac{1}{x}-\tfrac{1}{4}x)\,dx$$

$$= \int_0^1 \tfrac{3}{4}x\,dx + \int_1^2 (\tfrac{1}{x}-\tfrac{1}{4}x)\,dx$$

$$= \tfrac{3}{4}\cdot\tfrac{1}{2}x^2\Big|_0^1 + (\ln x - \tfrac{1}{4}\cdot\tfrac{1}{2}x^2)\Big|_1^2$$

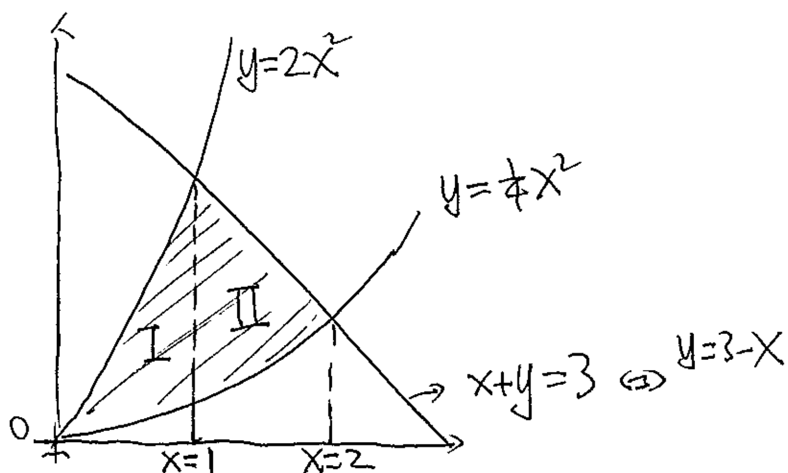
$$= \tfrac{3}{8}\cdot 1^2 - \tfrac{3}{8}\cdot 0^2 + (\ln 2 - \tfrac{1}{8}\cdot 2^2) - (\ln 1 - \tfrac{1}{8}\cdot 1)$$

$$= \tfrac{3}{8} - 0 + \ln 2 - \tfrac{1}{2} - 0 + \tfrac{1}{8} = \ln 2$$

28. $y=\frac{1}{4}x^2$, $y=2x^2$, $x+y=3$, $x\geq 0$

sln: $y=2x^2$ and $y=3-x$ intersecti at $2x^2=3-x$ $\Rightarrow x=1$

$y=\frac{1}{4}x^2$ and $y=3-x$ Intersect at $\frac{1}{4}x^2=3-x$ $\Rightarrow x=2$



		upper curve	lower curve
$[0,1]$	region I	$y=2x^2$	$y=\frac{1}{4}x^2$
$[1,2]$	region II	$y=3-x$	$y=\frac{1}{4}x^2$

$A = \text{area I} + \text{area II}$

$$= \int_0^1 (2x^2 - \tfrac{1}{4}x^2)\,dx + \int_1^2 (3-x) - \tfrac{1}{4}x^2\,dx$$

$$= \int_0^1 \tfrac{7}{4}x^2\,dx + \int_1^2 3-x-\tfrac{1}{4}x^2\,dx$$

$$= \tfrac{7}{4}\cdot\tfrac{1}{3}x^3\Big|_0^1 + \left(3x-\tfrac{1}{2}x^2-\tfrac{1}{4}\cdot\tfrac{1}{3}x^3\right)\Big|_1^2$$

$$= \tfrac{7}{12}\cdot 1^3 - 0 + (3\cdot 2-\tfrac{1}{2}\cdot 2^2-\tfrac{1}{12}\cdot 2^3)-(3\cdot 1-\tfrac{1}{2}\cdot 1^2-\tfrac{1}{12}\cdot 1^3)$$

$$= \tfrac{7}{12}+6-2-\tfrac{2}{3}-3+\tfrac{1}{2}+\tfrac{1}{12}$$

$$= 1+\tfrac{6}{12} = \tfrac{3}{2}$$

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