Homework 8 (due on 10/25)

- Read Sections 19 and 20 for the next week.
- We are going to have the second quiz on Monday, October 28. It covers the material we have learned up to the end of this week (10/18).

17.2 Let $f(x) = 4$ for $x \geq 0$, $f(x) = 0$ for $x < 0$, and $g(x) = x^2$ for all x. Thus $\text{dom}(f) = \text{dom}(g) = \mathbb{R}$.

(a) Determine the following functions: $f + g$, fg, $f \circ g$, $g \circ f$. Be sure to specify their domains.

(b) Which of the functions f, g, $f + g$, fg, $f \circ g$, $g \circ f$ is continuous?

17.3 Accept on faith that the following familiar functions are continuous on their domains: $\sin x$, $\cos x$, e^x, 2^x, $\log_e x$ for $x > 0$, x^p for $x > 0$ [p any real number]. Use these facts and theorems in this section to prove the following functions are also continuous. (b) $[\sin^2 x + \cos^6 x]^\pi$ (e) $\tan x$ for $x \neq$ odd multiple of $\frac{\pi}{2}$.

17.10 Prove the following functions are discontinuous at the indicated points. You may use either Definition 17.1 or the $\varepsilon - \delta$ property in Theorem 17.2.

(a) $f(x) = 1$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$, $x_0 = 0$;

(b) $g(x) = \sin(\frac{1}{x})$ for $x \neq 0$ and $g(0) = 0$, $x_0 = 0$;

(c) $\text{sgn}(x) = 1$ for $x > 0$, $\text{sgn}(x) = -1$ for $x < 0$, and $\text{sgn}(0) = 0$, $x_0 = 0$.

17.12 (a) Let f be a continuous real-valued function with domain (a, b). Show that if $f(r) = 0$ for each rational number r in (a, b), then $f(x) = 0$ for all $x \in (a, b)$.

(b) Let f and g be continuous real-valued functions on (a, b) such that $f(r) = g(r)$ for each rational number r in (a, b). Prove $f(x) = g(x)$ for all $x \in (a, b)$. Hint: Use part (a).

18.2 Reread the proof of Theorem 18.1 (a continuous function reaches max and min) with $[a, b]$ replaced by (a, b). Where does it break down? Discuss.

18.6 Prove $x = \cos x$ for some x in $(0, \frac{\pi}{2})$.

18.9 Prove that a polynomial function f of odd degree has at least one real root. Hint: You may assume that the leading term is $a_n x^n$, where n is the degree of the polynomial and $a_n > 0$. Then you may study the signs of $f(m)$ and $f(-m)$ for big $m \in \mathbb{N}$ by considering the limits of $f(m)/m^n$ and $f(-m)/m^n$ as $m \to \infty$.

E1 Let $f(x) = 0$ for all $x \in \mathbb{Q}$ and $f(x) = 1$ for all $x \in \mathbb{R} \setminus \mathbb{Q}$. Show that f is not continuous at any $x \in \mathbb{R}$. Hint: Use the denseness of $\mathbb{Q}$ (4.7) and the denseness of $\mathbb{R} \setminus \mathbb{Q}$ (Exercise 4.12).