

Homework 6 (due on 10/14 Monday)

- Read Sections 15 and 17 for the next week.
 - The first midterm is to be held on October 7. It covers everything we learned up to the end of the previous week. The exam contains 5 problems, which are similar to those in HW1-HW5. The class time will be extended for 10 minutes.
- 14.1 (e) Determine the convergence of $\sum \frac{\cos^2 n}{n^2}$ and justify your answer.
- 14.2 (a) Determine the convergence of $\sum \frac{n-1}{n^2}$ and justify your answer.
- 14.4 (c) Determine the convergence of $\sum \frac{n!}{n^n}$ and justify your answer. Hint: You may use the limit $(1 + \frac{1}{n})^n \rightarrow e \approx 2.71828$.
- 14.7 Prove that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges. Hint: You may use the fact that if $0 \leq a < 1$ and $p > 1$, then $a^p \leq a$.
- 14.13 (b) Prove $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. Hint: Use $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$.
- (c) Prove $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. Hint: Note $\frac{n-1}{2^{n+1}} = \frac{n}{2^n} - \frac{n+1}{2^{n+1}}$.
- (d) Use (c) to calculate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.