Homework 2 (due on 9/13)

Read Sections 5,7,8,9 for the next week.

3.3 Prove (iv) and (v) of Theorem 3.1.

(iv) $(-a)(-b) = ab$ for all a, b;

(v) $ac = bc$ and $c \neq 0$ imply $a = b$.

3.4 Prove (v) and (vii) of Theorem 3.2.

(v) $0 < 1$;

(vii) If $0 < a < b$, then $0 < b^{-1} < a^{-1}$.

Remark. For the above problems, your proofs should not use anything other than the axioms and the statements of Theorem 3.1/Theorem 3.2 that have been proved.

3.7 (a) Show $|b| < a$ if and only if $-a < b < a$.

(b) Show $|a - b| < c$ if and only if $b - c < a < b + c$.

(c) Show $|a - b| \leq c$ if and only if $b - c \leq a \leq b + c$.

4.5 Let S be a nonempty subset of $\mathbb{R}$ that is bounded above. Prove if $\sup S$ belongs to S, then $\sup S = \max S$. Hint: Your proof should be very short.

4.6 & 5.5 Let S be a nonempty subset of $\mathbb{R}$.

(a) Prove $\inf S \leq \sup S$. Note: You must consider all cases: S may or may not be bounded above, and may or may not be bounded below.

(b) What can you say about S if $\inf S = \sup S$?

E1 Show that for any finite nonempty set $S \subset \mathbb{R}$, $\max S$ exists. Hint: Prove by induction on $|S|$, i.e., the number of elements of S.

E2 Let $a, b \in \mathbb{R}$ and $a < b$. Show that $\inf(a, b] = a$.