GATEWAY EXAMS IN CALCULUS

at Michigan State University

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Gateway Exams are designed to test basic skills and knowledge that are needed later in a course or in ensuing courses. They in no way test understanding of concepts, which is done in other ways. The choice of problems is meant to be representative, not comprehensive.

The problems on such exams have surely been seen by students before and are of the type that students should have some facility with. If the students find there are certain types of problems they have trouble with, they are responsible for practicing sufficiently, on their own or with others, to gain the requisite skills. Suggested sources of explanations and practice problems are available.

The algebra exam (Calculus Gateway I) covers ten types of algebra and trigonometry problems that arise in calculus. For emphasis, these ten types do not entail a comprehensive coverage of all such skills; it is hoped facility with these will lead to facility with the rest. Following is a description of the types of problems and some hints about what we are expecting.

1. *Typical question*: Expand $\left(3xz-\frac{2y}{z}\right)^2$ or $\left(3xz-\frac{2y}{z}\right)\left(3xz+\frac{2y}{z}\right)$.

Comments: You should have memorized $(u+v)^2=u^2+2uv+v^2$ and $(u-v)(u+v)=u^2-v^2$. (In particular, doing these problems by "FOIL" is mathematically correct but *much* slower. In calculus, such a computation would be a small part of a big problem that you definitely do not want to make longer than you have to.) For this problem, $\left(3xz-\frac{2y}{z}\right)^2=(3xz)^2-2(3xz)\left(\frac{2y}{z}\right)+\left(\frac{2y}{z}\right)^2=9x^2z^2-12xy+4\frac{y^2}{z^2}$ and $\left(3xz-\frac{2y}{z}\right)\left(3xz+\frac{2y}{z}\right)=(3xz)^2-\left(\frac{2y}{z}\right)^2=9x^2z^2-4\frac{y^2}{z^2}$. Finally, you should be encouraged to do the middle steps in your head.

2. *Typical question*: Simplify $\sqrt[5]{x\sqrt{x^3}}$.

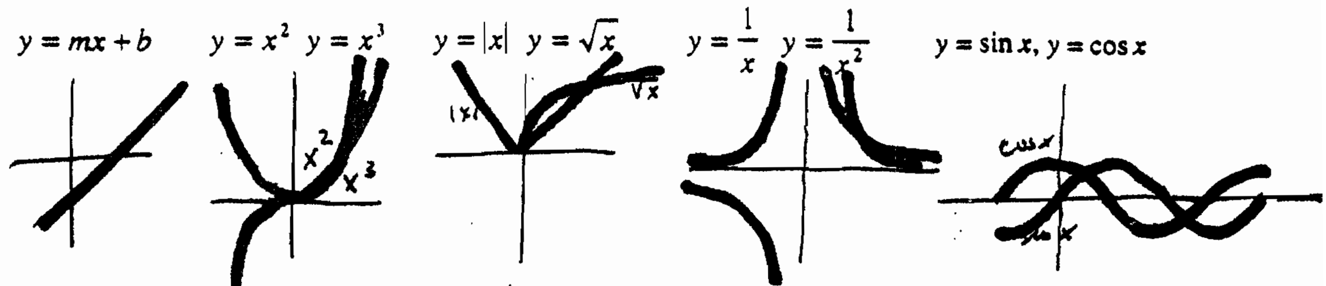
Comments: Rational exponents come up often enough in calculus that you need facility with them. Here, $\sqrt[5]{x\sqrt{x^3}}=\left[x\left(x^3\right)^{1/2}\right]^{1/5}=\left[xx^{3/2}\right]^{1/5}=\left[x^{5/2}\right]^{1/5}=x^{1/2}=\sqrt{x}$.

3. *Typical question*: Factor x^2-6x-7. Solve: $x^2=6x+7$.

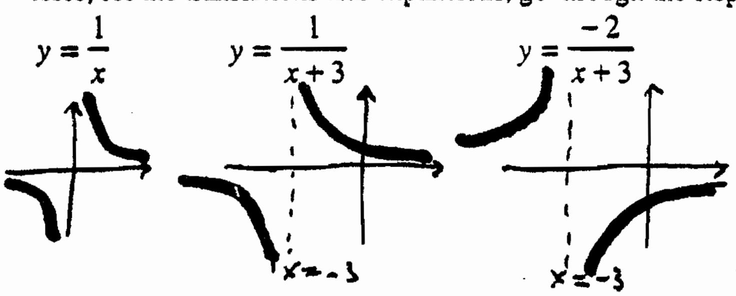
Comments: So many calculus, engineering, economics, etc., problems have simple quadratic problems embedded in them that you should be able to do simple ones by hand. Here, $x^2-6x-7=(x-7)(x+1)$ and $x^2=6x+7$, $x^2-6x-7=0$, $(x-7)(x+1)=0$, $x=-1,7$.

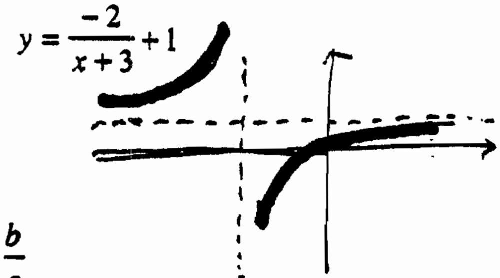
4. *Typical question*: Sketch the graphs of $y = e^x$ and $y = \dfrac{-2}{x+3} + 1$.

Comments: Even though you have a calculator handy, you should have memorized addition and multiplication tables. For similar reasons, even though you have a graphing calculator handy, there are several functions which are so common that you should have images in your mind of the basic shapes of their graphs, and also know simple translations and expansions. You should know the graphs of:



Here, for the translations and expansions, go through the steps:





5. *Typical question*: Simplify as much as possible: $\dfrac{\frac{a}{b} - \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}}$.

Comments: One way to simplify fractions like this is to multiply numerator and denominator by the lowest common denominator.

$$\frac{\frac{a}{b} - \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}} \cdot \frac{\frac{ab}{1}}{\frac{ab}{1}} = \frac{\frac{a}{b}\frac{ab}{1} - \frac{b}{a}\frac{ab}{1}}{\frac{1}{b}\frac{ab}{1} - \frac{1}{a}\frac{ab}{1}} = \frac{a^2 - b^2}{a-b} = \frac{(a-b)(a+b)}{a-b} = a + b$$

6. *Typical question*: Simplify as much as possible: $a - (1 - \{a - [a - (a-1)]\})$.

Comments: Be careful of negative signs as you clear parentheses and brackets. Here,
$a - (1 - \{a - [a - (a-1)]\}) = a - 1 + \{a - [a - a + 1]\} = a - 1 + a - [1] = 2a - 2$

7. *Typical question*: Find the point-slope equation for the following straight lines:

(a) The line through the points (-2, 3) and (4, 1).

(b) The line through the point (-2, 3) and parallel to the line $y = -3x + 7$.

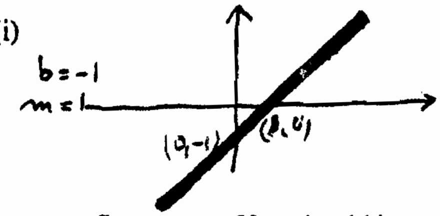
(c) The line through the point (-2, 3) and perpendicular to the line $y = -3x + 7$.

Comments: You should know $y - y_0 = m(x - x_0)$ is the point-slope equation for a straight line. Here, (x_0, y_0) is any point on the line and m = slope = $\dfrac{\textit{change in } y}{\textit{change in } x}$.

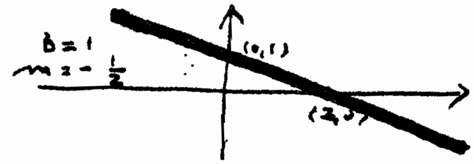
For (a), $m = \dfrac{\textit{change in } y}{\textit{change in } x} = \dfrac{1-3}{4-(-2)} = -\dfrac{1}{3}$ and you can use either point. Thus two possible answers are $y-3=-\frac{1}{3}(x+2)$ or $y-1=-\frac{1}{3}(x-4)$. For (b) and (c) you should know the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals. Thus the answer for (b) is $y-3=-3(x+2)$ and the answer for (c) is $y-3=\frac{1}{3}(x+2)$.

8. *Typical question*: The graph below represents a straight line with equation $y = mx + b$. Estimate m and b.

(i)



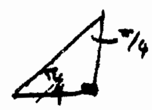
(ii)

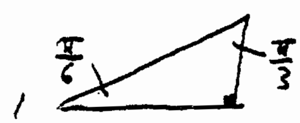


Comments: You should know m = slope = $\dfrac{\textit{change in } y}{\textit{change in } x}$ and b = y-intercept. Many of the calculus problems that you will see will require you to estimate slopes and sometimes also estimate b. Here, for (i) $m \approx 3$ and $b \approx -1$ and for (ii) $m \approx -1/3$ and $b \approx 1$.

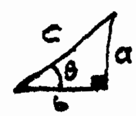
9. *Typical question*: Sketch a $30^0 - 60^0$ right triangle and label its sides. Use it to find $\cos\frac{\pi}{6}$ and $\csc\frac{\pi}{6}$.

Comments: You should know the triangles





and that in a right triangle



$\sin\theta = \frac{a}{c}$, etc. Thus $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\csc\frac{\pi}{6} = \frac{2}{1} = 2$.

10. *Typical question*: Sketch the graph of $y = \sin x$ over the interval $= \frac{1}{2}\pi \le x \le \frac{5}{2}\pi$. Find $\sin(-\frac{1}{2}\pi)$ and $\sin\frac{3}{2}\pi$.

Comments: You should know the graphs of $y = \sin x$, $y = \cos x$, $y = \tan x$ (as well as $y = \cot x$, $y = \sec x$, $y = \csc x$) over intervals bigger than just $[0, 2\pi]$. Using these graphs is one way of easily finding some values of the trigonometric functions. For example, you can easily see from the graph that $\sin(-\frac{1}{2}\pi) = -1$ and $\sin\frac{3}{2}\pi = -1$