Name (Print Clearly): Student Number:

MTH132 Section 1 & 12, Test 4

April 27, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [5 pts each]. Find the derivatives (FTC I).

(a) $y = \int_0^x \tan(t^2)dt$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$y' = \tan(x^2)$$

(b) $y = \int_{x^2}^0 \sqrt{t}(1+t^2)dt$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\cdot b'(x) - f(a(x))\cdot a'(x)$$

$$y' = 0 - \sqrt{x^2}(1+x^4)\cdot 2x = -2x\cdot\sqrt{x^2}(1+x^4)$$

$$(\sqrt{x^2} = |x|)$$

2. [6 pts each]. Evaluate indefinite integrals.

(a) $\int 6x\sqrt{1+3x^2}\,dx$

let $t = 1+3x^2$, $dt = 6x\,dx$

$$= \int \sqrt{t}\,dt$$

$$= \frac{2}{3}t^{3/2} + C$$

$$= \frac{2}{3}(1+3x^2)^{3/2} + C$$

(b) $\int \frac{x^3+\sqrt{x}}{x^2}dx$

$$= \int x + x^{-3/2}\, dx = \int x\,dx + \int x^{-3/2} dx$$

$$= \tfrac{1}{2}x^2 + (-2)x^{-1/2} + C$$

3. [7 pts each]. Evaluate the definite integrals.

(a) $\int_0^1 (1+x^3)dx$

$$= \left[x + \tfrac{1}{4}x^4\right]_0^1 = \left(1+\tfrac{1}{4}\right) - 0 = \frac{5}{4}$$

(b) $\int_{-\pi/2}^{\pi/2} (\sin^4 x \cdot \cos x)dx$ let $t = \sin x$, $dt = \cos x\, dx$

$$= \int_{-1}^{1} t^4\, dt = \tfrac{1}{5}t^5 \Big|_{-1}^{1} = \tfrac{1}{5}\left(1^5 - (-1)^5\right) = \frac{2}{5}$$

or $= 2\int_0^1 t^4 dt = 2 \cdot \frac{1}{5} \cdot t^5 \Big|_0^1 = \frac{2}{5}$ (t^4 even func)

$\sin x$ is odd, but $\sin^4 x = (\sin x)^4$ is even!

so $\sin^4 x \cdot \cos x$ is even!

4. [12 pts]. Let $f(x) = x^2 + 2x - 3$, $-3 \le x \le 3$.

(a) Compute $\int_{-3}^{3} f(x)dx$

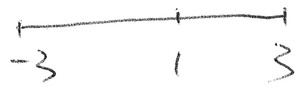
$$= \left[\tfrac{1}{3}x^3 + x^2 - 3x\right]_{-3}^{3} = \left[\tfrac{1}{3}27 + 9 - 9\right] - \left[-\tfrac{27}{3} + (-3)^2 - 3\cdot(-3)\right]$$

$$= 9 - 9 = 0.$$

(b) Find the total area enclosed by x-axis and the graph of $f(x)$ over $[-3,3]$

$f(x)$ can be negative over $[-3, 3]$.

$f(x) = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow x = 1, -3.$

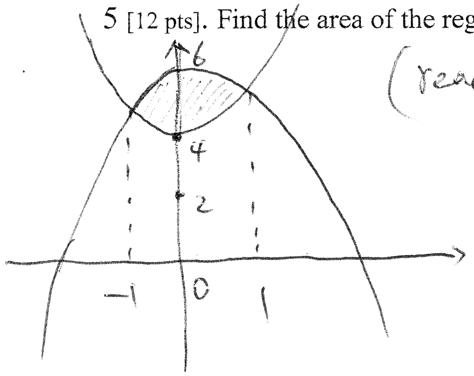


$$A_1 = \int_{-3}^{1} f(x)dx = \left[\tfrac{1}{3}x^3 + x^2 - 3x\right]_{-3}^{1} = -\frac{32}{3}$$

$$A_2 = \int_{1}^{3} f(x)dx = \left[\tfrac{1}{3}x^3 + x^2 - 3x\right]_{1}^{3} = \frac{32}{3}$$

$$\text{Total Area} = |A_1| + |A_2| = \frac{64}{3}$$

5 [12 pts]. Find the area of the region enclosed by the curves $y = x^2 + 4$ and $y = 6 - x^2$.



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upper curve : $y = 6 - x^2$

lower curve : $y = x^2 + 4$

$$\text{so Area} = \int_{-1}^{1} [(6 - x^2) - (x^2 + 4)]dx$$

$$= \int_{-1}^{1} 2 - 2x^2 \, dx$$

$$= \left[2x - \tfrac{2}{3}x^3\right]_{-1}^{1} = \left(2 - \tfrac{2}{3}\right) - \left(-2 + \tfrac{2}{3}\right)$$

$$= 4 - \frac{4}{3} = \frac{8}{3}$$

6. [10 pts] Set up a Riemann sum approximation to the integral $\int_0^2 (x^2+1)dx$ by partitioning the interval $[0,2]$ into 4 subintervals of equal length and using the left endpoint of each subinterval.

(a) Evaluate this sum.

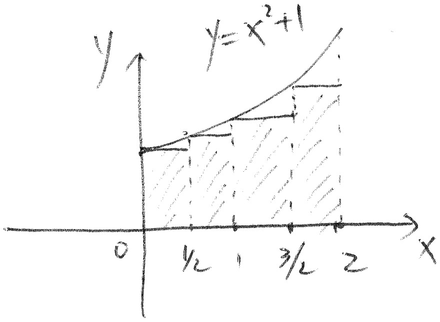
$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}, \quad x_k = a + k\cdot\Delta x = 0 + k\cdot\frac{1}{2} = \frac{k}{2}$$

$$c_k = x_{k-1} = \frac{k-1}{2}. \quad f(c_k) = \left(\frac{k-1}{2}\right)^2 + 1$$

$$S = \sum_{k=1}^{4} f(c_k)\cdot\Delta x = \sum_{k=1}^{4}\left[\left(\frac{k-1}{2}\right)^2+1\right]\cdot\frac{1}{2} = \frac{1}{2}\left(1+\frac{5}{4}+2+\frac{13}{4}\right)$$

$$= \frac{1}{2}\left(3+\frac{9}{2}\right) = \frac{15}{4}$$

(b) Sketch the graph of $y = x^2+1$ over $[0,2]$. Sketch (shade) the rectangles associated to this Riemann Sum.



(c) [bonus 2pts]. Express $\lim_{x\to\infty}\sum_{k=1}^{n}[\frac{2k}{n}+1]\cdot\frac{2}{n}$ as a definite integral.

$$\frac{1}{n} \to dx, \quad \frac{k}{n} \to x, \quad [a,b] = [0,1]$$

$$\int_0^1 (2x+1)\cdot 2\cdot dx = \int_0^1 (4x+2)\,dx$$