Name (Print Clearly): Student Number:

MTH132 Section 1 & 12, Test 3

April 3, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [3 pts each]. Find limits of each of the following functions.

(a) $\lim_{x\to1}\frac{x^{3/5}-1}{x^{1/3}-1}$

$$\overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to1}\frac{\frac{3}{5}X^{-2/5}}{\frac{1}{3}\cdot X^{-2/3}} \overset{DSP}{=} \frac{3}{5}\cdot\frac{3}{1} = 9/5$$

(b) $\lim_{x\to0} x\csc(2x)$

$$= \lim_{X\to0} X\cdot\frac{1}{\sin 2X} = \lim_{X\to0}\frac{X}{\sin 2X} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{X\to0}\frac{1}{\cos 2X\cdot 2} \overset{DSP}{=} \frac{1}{2}$$

$(\cos 0 = 1)$

(c) $\lim_{x\to\infty}\frac{x\sqrt{x}-2x^2}{3x^2+\sin x}$ divide by x^2

$$= \lim_{X\to\infty}\frac{\frac{1}{\sqrt{X}}-2}{3+\frac{\sin X}{X^2}} = \frac{0-2}{3+0} = -\frac{2}{3}$$

$\lim_{X\to\infty}\frac{\sin X}{X^2} = 0$, by Sandwich Theorem.

(d) $\lim_{X\to1^+}\frac{1-X}{|X-1|}$

$X > 1$

$$= \lim_{X\to1^+}\frac{1-X}{X-1} = \lim_{X\to1^+} -1 = -1$$

2. [5 pts each]. Find indefinite integrals.

(a) $\int(x^{-1/3}+3x^2-2\cos x)dx$ $= \int x^{-1/3}dx + \int 3x^2 dx - \int 2\cdot\cos x\, dx$

$= \frac{1}{-\frac{1}{3}+1} x^{2/3} + 3\cdot\frac{1}{2+1}\cdot x^3 - 2\cdot\sin x + C$

$= \frac{3}{2}x^{2/3} + x^3 - 2\sin x + C$

① ① ① ①

(b) $\int\frac{1}{(2r+5)^2}dr$ let $t = 2r+5$ ① $dt = 2dr \Rightarrow dr = \frac{1}{2}dt$ ①

$= \int \frac{1}{t^2}\cdot\frac{1}{2}dt = \frac{1}{2}\int t^{-2}dt$ ① $= -\frac{1}{2}t^{-1} + C$ ① $= -\frac{1}{2}(2r+5)^{-1}+C$ ①

(substitution)

3. [8 pts]. Solve initial value problem: $\frac{dy}{dx} = \sin(\pi\cdot x), y(0)=1$.

$y = \int \sin(\pi x)dx = -\frac{1}{\pi}\cos(\pi x) + C$ ⑤

$\because y(0)=1, \quad -\frac{1}{\pi}\cos(\pi\cdot 0) + C = -\frac{1}{\pi}\cos 0 + C = -\frac{1}{\pi} + C = 1$

$\therefore C = 1 + \frac{1}{\pi}$. ③

So $y(x) = -\frac{1}{\pi}\cos(\pi x) + 1 + \frac{1}{\pi}$

4. [10 pts]. Let $f(x) = x + \frac{1}{x}, \quad [\frac{1}{2}, 2]$.

(a) Show that f(x) satisfies the hypotheses of the Mean Value Theorem over the given interval.

x is cont. and diff. on $\mathbb{R}$

$\frac{1}{x}$ is cont. and diff on $(0, \infty)$ ④

So $f(x) = x + \frac{1}{x}$ is cont and. diff on $[\frac{1}{2}, 2]$

(b) Find c which satisfies the equation of the Mean Value Theorem: $\frac{f(b)-f(a)}{b-a} = f'(c)$, on $[\frac{1}{2}, 2]$.

$\frac{f(2)-f(\frac{1}{2})}{2-\frac{1}{2}} = f'(c) \Rightarrow \frac{(2+\frac{1}{2})-(\frac{1}{2}+2)}{2-\frac{1}{2}} = 0 = 1 - \frac{1}{c^2} \Rightarrow c = 1 \in [\frac{1}{2}, 2]$

③ ③

$(f'(x) = 1 - \frac{1}{x^2})$

($c=-1$, is not in this interval)

5 [12 pts]. Let $f(x)=\dfrac{x^2+3x}{x-1}$ (Note that $f'(x)=\dfrac{x^2-2x-3}{(x-1)^2}$ and $f''(x)=\dfrac{8}{(x-1)^3}$)

(a) Find the intervals where f is increasing or decreasing.

critical pts: $f'(x)=0 \Rightarrow x^2-2x-3=0 \Rightarrow x=-1,\ 3.$ ①

$f'(x)$ is undefined $\Rightarrow x=1.$

sign of f': $+$ (−1) $-$ (1) $-$ (3) $+$

f is increasing on $(-\infty,-1)$ and $(3,\infty)$ ①

f is decreasing on $(-1,1)$ and $(1,3)$ ①

(b) Find the intervals where f is concave up or concave down.

$f''>0$ when $x>1$, $f''<0$ when $x<1$

sign of f'': $-$ (1) $+$ ①

so f is concave up on $(1,\infty)$ ①

f is concave down on $(-\infty,1)$ ①

(c) Find all local extrema (types and values of x and the function).

f' changes sign from "+" to "−" at -1,

so f has local maximum value $f(-1)=1$ at $x=-1$; 0.5 0.5 0.5

f' changes sign from "−" to "+" at 3.

so f has local minimum value $f(3)=9$ at $x=3$. 0.5 0.5 0.5

(d) Find the equations of all asymptotes (vertical and oblique asymptotes).

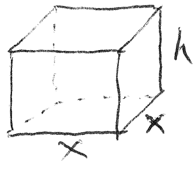
vertical asymptote: $x=1$, ① since $\lim\limits_{x\to1^+}f(x)=\infty$ and $\lim\limits_{x\to1^-}f(x)=-\infty$

$$\begin{array}{r|l} & x+4 \\ \hline x-1 & x^2+3x \\ & \underline{x^2-x} \\ & 4x \\ & \underline{4x-4} \\ & 4 \end{array}$$

So $f(x)=x+4+\dfrac{4}{x-1}$

oblique asymptote: $y=x+4$. ②

6 [10 pts]. A rectangular box with volume 36 cubic centimeters is to be built with a square base and top. The material used for the **base** costs \$5 per square centimeter while the material used for the **side** panels and **top** costs \$3 per square centimeter. Find the minimum cost to build such a box. ①



given info: $36=x^2\cdot h \Rightarrow h=\dfrac{36}{x^2}$ ①

target: min $Cost=\$5\cdot x^2+\$3\cdot 4xh+\$3\cdot x^2$ ②

$Cost=8\cdot x^2+12xh=8x^2+12x\cdot\dfrac{36}{x^2}$ ①

$C(x)=8x^2+\dfrac{12\cdot 36}{x}$, dom $=(0,\infty)$

$C'=16x-\dfrac{12\times 36}{x^2}$ ②

$c' = 16x - \frac{12 \cdot 36}{x^2} = 0 \Rightarrow$ critical pts : $x^3 = \frac{12 \cdot 36}{16} = \frac{3 \cdot 4 \cdot 4 \cdot 9}{4 \cdot 4}$

$\Rightarrow x^3 = 3^3 \Rightarrow x = 3$ ①   − ① + sign of c'

0 3

so $C(x)$ obtains local (global) minimum value $C(3) = 216$ ①

when $x = 3$, $h = \frac{36}{9} = 4$.

7. Use Newton's Method to approximate $\sqrt{3}$.

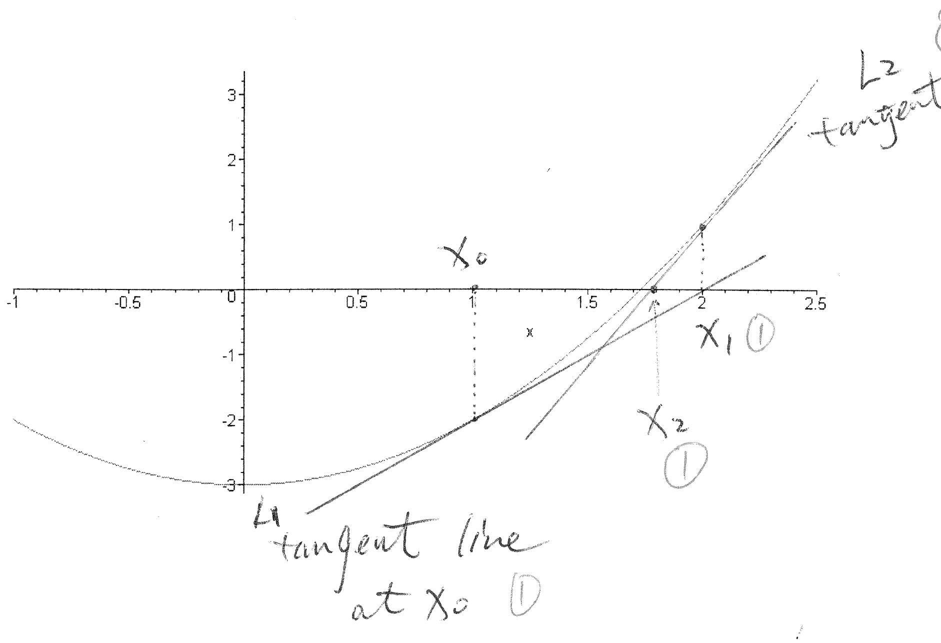
(a) [8 pts] Construct a function $f(x)$ such that $\sqrt{3}$ is a root of $f(x)=0$. Suppose $x_0 = 1$ then use Newton's method to construct x_1 and x_2. let $x = \sqrt{3}$ then $x^2 = 3 \Rightarrow x^2 - 3 = 0$ ②

$f(x) = x^2 - 3$. $f' = 2x$. $x_0 = 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1-3}{2} = 2$. ③

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{4-3}{4} = 1.75$ ③

(b)[bonus 4 pts] Complete the sketch including labels to graphically illustrate Newton's Method and the determination of x_1 and x_2.



x_1 is the intersection of L_1 and x-axis.

x_2 is the intersection of L_2 and x-axis.