Name (Print Clearly): ________________ PID: ________________

MTH133 Section 64, Test4

Dec. 8, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [5 pts each]. Test for convergence. **If you encounter convergent geometric series, please find the sum.**

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+2}{n^2-n+1}$

$b_n = \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$ $\frac{3}{2} > 1$

So $\sum b_n$ converges $\Rightarrow \sum a_n$ converges.

(b) $\sum_{n=2}^{\infty} \frac{2^{n+1}-5}{4^n}$

$= \sum_{n=2}^{\infty} \frac{2^{n+1}}{4^n} - \sum_{n=2}^{\infty} \frac{5}{4^n}$

$= \frac{\frac{1}{2}}{1-\frac{1}{2}} - \frac{5/16}{1-\frac{1}{4}}$

$= 1 - \frac{5}{12} = \frac{7}{12}$

(c) $\sum_{n=1}^{\infty}(1-\frac{1}{n})^n$

$\lim_{n\to\infty}(1-\frac{1}{n})^n = e^{-1} \neq 0$ ∴ diverges

(d) $\sum_{n=1}^{\infty}\frac{(-3)^n}{n!}$

soln ①. alternating series test

soln ②. $\sum |a_n|$, ratio test : $\frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{3\cdot n!}{(n+1)n!} = \frac{3}{n+1} \to 0 < 1$

It converges.

2[6 pts]. Express the number $0.\overline{314} = 0.314314314\cdots$ as a rational number.

$\frac{314}{999}$ (geometric series)

3[6 pts]. Find the Taylor polynomial of order 3 for the function $f(x) = \sin x$ at $x = \pi/2$.

$f(x) = \sin x$		1
f'	$\cos x$	0
f''	$-\sin x$	-1
f'''	$-\cos x$	0

$P_3 = 1 - \frac{1}{2!}(x - \pi/2)^2$

(degree 3 term is 0)

4[8pts].Find the interval of convergence (include the endpoints) of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(3x-1)^n$.

$|3x-1| < 1 \iff |x - \frac{1}{3}| < \frac{1}{3}$

$0 \quad 1/3 \quad 2/3$

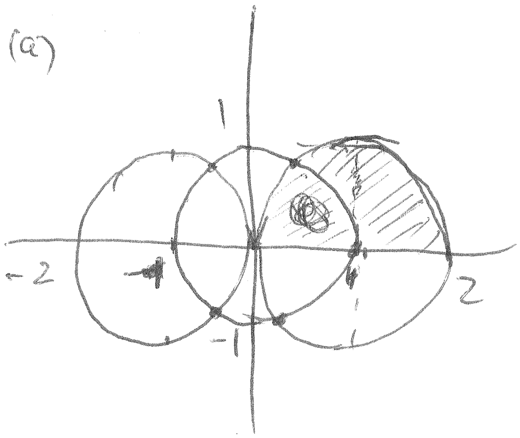
At $x=0$, $\sum \frac{(-1)^{2n-1}}{n} = \sum \frac{-1}{n}$ diverges.

At $x = 2/3$, $\sum \frac{(-1)^{n-1}}{n}$ converges.

So the interval is $(0, \frac{2}{3}]$, $0 < x \leq \frac{2}{3}$

5. (a) [6 pts] Draw the curves given in Polar coordinates by $r = 1 + \cos 2\theta$ and $r = 1$ (Hint: use the symmetry about x-axis, y-axis and the origin). (b) [4 pts] Find their intersection points. (c) [2 pts] Find the slope of the first curve at $\theta = \pi/4$. (d) [8 pts] Find the area of the region inside the first curve and outside the second curve.

(a)



(b) $\begin{cases} r = 1 + \cos 2\theta \\ r = 1 \end{cases} \Rightarrow \cos 2\theta = 0$

$\Rightarrow \theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}.$

(c). $\text{slope} = \frac{(r\cdot\sin\theta)'}{(r\cos\theta)'} = \left.\frac{f'(\theta)\cdot\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cdot\cos\theta - f(\theta)\sin\theta}\right|_{\theta=\pi/4}$

$= \left.\frac{-2\sin 2\theta\cdot\sin\theta + (1+\cos 2\theta)\cdot\cos\theta}{-2\sin 2\theta\cdot\cos\theta - (1+\cos 2\theta)\sin\theta}\right|_{\theta=\frac{\pi}{4}} = \frac{-2\cdot\frac{\sqrt{2}}{2} + 1\cdot\frac{\sqrt{2}}{2}}{-2\frac{\sqrt{2}}{2} - 1\cdot\frac{\sqrt{2}}{2}}$

$= \frac{-\sqrt{2} + \frac{\sqrt{2}}{2}}{-\sqrt{2} - \frac{\sqrt{2}}{2}} = \frac{-\frac{1}{2}\sqrt{2}}{-\frac{3}{2}\sqrt{2}} = \frac{1}{3}$

(d) $4\cdot\int_0^{\pi/4} \frac{1}{2}\left((1+\cos 2\theta)^2 - 1\right) d\theta$

$= 2\cdot\int_0^{\pi/4} (1 + 2\cos 2\theta + \cos^2 2\theta - 1)\, d\theta$

$= 2\cdot\int_0^{\pi/4} \left(2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right) d\theta$

$= 2\cdot\left(\sin 2\theta + \frac{1}{2}\theta + \frac{\sin 4\theta}{8}\right)\Big|_0^{\pi/4}$

$= 2\cdot\left(1 + \frac{\pi}{8} + 0\right) - 2\cdot 0 = 2 + \frac{\pi}{4}.$