Name (Print Clearly): ________________ PID: ________________

MTH133 Section 64, Test2

Oct.13, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [5 pts each]. Which of the following functions grows faster as $x \to \infty$?

(a) $f(x) = \sqrt{2x^2 - 5x}$, $g(x) = \ln x$.

consider $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\sqrt{2x^2-5x}}{\ln x} = \lim_{x\to\infty} \frac{\sqrt{2x^2}}{\ln x}$.

$= \lim_{x\to\infty} \frac{\sqrt{2}\cdot x}{\ln x} = \infty$, (see Ex 7.13 (d) in the lecture notes)

So $f(x)$ grows faster.

(b) $f(x) = x$, $g(x) = \dfrac{1}{\sin^{-1}(\frac{1}{x})}$.

$\lim_{x\to\infty} \frac{x}{\frac{1}{\sin^{-1}(\frac{1}{x})}} = \lim_{x\to\infty} x \cdot \sin^{-1}(\frac{1}{x}) = \lim_{x\to\infty} \frac{\sin^{-1}(\frac{1}{x})}{\frac{1}{x}}$

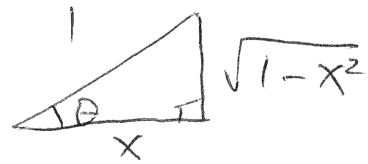
$\overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{1}{\sqrt{1-(1/x)^2}}\cdot(\frac{1}{x})'}{(\frac{1}{x})'} = \lim_{x\to\infty} \frac{1}{\sqrt{1-\frac{1}{x^2}}} = \frac{1}{\sqrt{1-0}} = 1$.

So f and g grow at the same rate,

2[5 pts each]. Find the derivatives (don't simplify).

(a) $y = \cos(\sin^{-1}(\sqrt{1-x^2}))$

let $\theta = \sin^{-1}\sqrt{1-x^2}$



so $\cos\theta = x$

$y = \cos\theta = x \Rightarrow y' = 1$.

(b) $y = \ln(\cosh x) - \frac{1}{2}\tanh^2 x$

$y' = \frac{1}{\cosh x} \cdot \sinh x - \tanh x \cdot \operatorname{sech}^2 x$

3 [5 pts each]. Evaluate the following integrals.

(a) $\int \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$ let $u = \tan^{-1} x$, $du = \frac{1}{1+x^2} dx$

$= \int \sqrt{u}\, du$

$= \frac{2}{3} u^{3/2} + C = \frac{2}{3}(\tan^{-1} x)^{3/2} + C$.

(b) $\int \frac{1-x}{\sqrt{1-x^2}}dx$

$$= \int \frac{1}{\sqrt{1-x^2}}dx - \int \frac{x}{\sqrt{1-x^2}}dx$$

let $u = 1-x^2$, $du = -2x\,dx$

$$= \sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{u}}du$$

$$= \sin^{-1}x + u^{1/2} + C$$

$$= \sin^{-1}x + \sqrt{1-x^2} + C$$

(c) $\int (x+2)^3 \cos x\,dx$

$$= (x+2)^3 \sin x + 3(x+2)^2 \cos x - 6(x+2)\sin x - 6\cos x + C$$

$(x+2)^3$	$+$	$\cos x$
$3(x+2)^2$	$-$	$\sin x$
$6(x+2)$	$+$	$-\cos x$
6	$-$	$-\sin x$
0		$\cos x$

(d) $\int \frac{x+4}{x^2+5x-6}dx$ (use partial fractions)

$$x^2+5x-6 = (x+6)(x-1)$$

$$\frac{x+4}{(x+6)(x-1)} = \frac{A}{x+6} + \frac{B}{x-1}$$

$$A = \left.\frac{x+4}{x-1}\right|_{x=-6} = \frac{2}{7}$$

$$B = \left.\frac{x+4}{x+6}\right|_{x=1} = \frac{5}{7}$$

$$= \int \left(\frac{2/7}{x+6} + \frac{5/7}{x-1}\right)dx$$

$$= \frac{2}{7}\ln|x+6| + \frac{5}{7}\ln|x-1| + C$$

4 (a) [5pts]. Factorize $t^4 - 3t^2 + 2$ (Hint: it has 4 distinct linear factors)

$$= (t^2-2)(t^2-1) = (t-\sqrt{2})(t+\sqrt{2})(t-1)(t+1)$$

(b) [5pts] Find the partial fractions of $\dfrac{t}{t^4 - 3t^2 + 2}$.

$$\frac{t}{t^4-3t^2+2} = \frac{A}{t-\sqrt{2}} + \frac{B}{t+\sqrt{2}} + \frac{C}{t-1} + \frac{D}{t+1}$$

$$A = \frac{t}{(t+\sqrt{2})(t-1)(t+1)}\bigg|_{t=\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}\cdot 1} = \frac{1}{2}$$

$$B = \frac{t}{(t-\sqrt{2})(t^2-1)}\bigg|_{t=-\sqrt{2}} = \frac{-\sqrt{2}}{-2\sqrt{2}\cdot 1} = \frac{1}{2}$$

$$C = \frac{t}{(t^2-2)\cdot(t+1)}\bigg|_{t=1} = \frac{1}{-1\cdot 2} = -\frac{1}{2}, \quad D = \frac{t}{(t^2-2)}$$

5. (a) [5pts]. Rewrite $(\sinh x + \cosh x)^3$ in terms of exponential functions.

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2} \Rightarrow \sinh x + \cosh x = e^x$$

So it is e^{3x}

(b) [5pts]. Evaluate $\int_0^{\ln 3} (\sinh x + \cosh x)^3 \, dx$

$$= \int_0^{\ln 3} e^{3x} dx \qquad \text{let } u = 3x. \quad du = 3dx$$

$$= \int_0^{3\ln 3} e^u \cdot \frac{1}{3} du = \frac{1}{3} e^u \Big|_0^{3\ln 3} = \frac{1}{3} e^{3\ln 3} - \frac{1}{3} e^0$$

$$= \frac{1}{3} e^{\ln 27} - \frac{1}{3} = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$$