Name (Print Clearly): ________________ Student Number: ________________

MTH133 Section 64, Test1

Sept.22, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [10 pts]. A spring has natural length 10 cm. An 80 kg force stretches the spring to 18 cm.

(a) What is the spring constant k?

$F = kx$, $F = 80kg$, $x = 18 - 10 = 8\,cm$

So $k = \frac{F}{x} = \frac{80}{8} = 10\ kg/cm$

(b) How large is the force to stretch the spring to 36cm.?

$F = kx$, $k = 10$, $x = 36 - 10 = 26$.

So $F = 10 \cdot 26 = 260\ kg$.

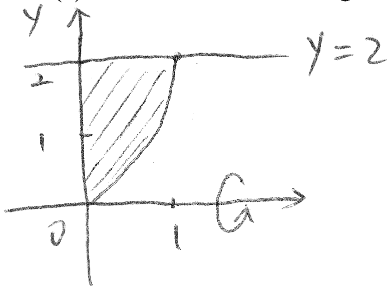
(c) How much work is done in stretching the spring from its natural length to 15cm?

$W = \int_a^b F(x)\,dx = \int_0^5 kx \cdot dx = 10\int_0^5 x \cdot dx$

$= 10 \cdot \frac{1}{2}x^2\Big|_0^5 = 125\ kg \cdot cm$

2 [10 pts]. The region is in the first quadrant bounded above by the line y = 2, and below by the curve $y = 2x^{3/2}, 0 \le x \le 1$.

(a) Find the area of the region.



$$A = \int_0^1 (2 - 2x^{3/2})\,dx$$

$$= \left[2x - 2\cdot\frac{2}{5}x^{5/2}\right]_0^1 = 2 - \frac{4}{5} = 6/5$$

(b) Find the volume of the solid which is generated by revolution of the region about x-axis.

$$V = \int_0^1 \pi\left[R(x)^2 - r(x)^2\right] dx$$

$$= \int_0^1 \pi\left(2^2 - (2x^{3/2})^2\right) dx = \int_0^1 \pi(4 - 4x^3)\,dx$$

$$= \left[4x\pi - \pi x^4\right]_0^1 = 4\pi - \pi = 3\pi$$

(c) Find the perimeter of this region.

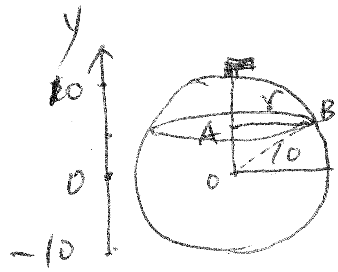
perimeter of this region $= 2 + 1 + L$ $\qquad t = 1 + 9x$

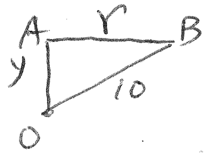
$$L = \int_0^1 \sqrt{1 + (f')^2}\,dx = \int_0^1 \sqrt{1 + (3x^{1/2})^2}\,dx = \int_0^1 \sqrt{1 + 9x}\,dx$$

$$= \int_1^{10} \sqrt{t}\cdot\frac{1}{9}\,dt = \frac{1}{9}\cdot\frac{2}{3}\cdot t^{3/2}\Big|_1^{10} = \frac{2}{27}\left(\sqrt{1000} - 1\right)$$

So Perimeter $= 3 + \frac{2}{27}\left(\sqrt{1000} - 1\right)$

3 [10 pts]. A spherical tank measures 20 ft in diameter. It is full of gasoline weighting 50 lb/ ft^3. How much work does it take to pump the gasoline to the top of the tank?





$F = \text{weight} = \Delta V \cdot 50$

$= \pi \cdot r^2 \cdot \Delta y \cdot 50 = \pi \cdot (100 - y^2) \cdot \Delta y \cdot 50$

$\Delta W = F \cdot d = 50\pi (100 - y^2) \cdot \Delta y \cdot (10 - y)$

$$W = \int_{-10}^{10} 50\pi (100 - y^2)(10 - y) \cdot dy .$$

$$= 50\pi \int_{-10}^{10} (1000 - 10y^2 - 100y + y^3)\, dy \quad \text{(by symmetry)}$$

$$= 100\pi \int_{0}^{10} (1000 - 10y^2)\, dy = 100\pi \cdot \left[1000y - \frac{10}{3} y^3\right]_0^{10}$$

$$= 100\pi \cdot \frac{20000}{3}$$

4[10 pts]. Evaluate the integral.

(a) $\int \cot x dx$.

$$= \int \frac{\cos x}{\sin x} dx = \int \frac{(\sin x)'}{\sin x} dx = \ln|\sin x| + C$$

(b) $\int \frac{dx}{e^x + 2}$. (Hint: find $\int \frac{e^x}{e^x + 2} dx$. first)

$$I = \int \frac{1}{e^x + 2} dx = \int \frac{e^x + 2 - 1 - e^x}{e^x + 2} dx = \int \left(1 - \frac{1}{e^x + 2} - \frac{e^x}{e^x + 2}\right) dx$$

$$= x - I - \int \frac{e^x}{e^x + 2} dx = x - I - \ln(e^x + 2) + C$$

$$\Rightarrow 2I = x - \ln(e^x + 2) + C$$

$$\Rightarrow I = \frac{1}{2}[x - \ln(e^x + 2)] + C .$$

5[10 pts]. Find the derivatives (don't simplify).

(a) $y = \ln(\sin^2\theta)$

$$y = 2\ln(\sin\theta) \Rightarrow y' = 2\frac{1}{\sin\theta}\cdot\cos\theta$$

(b) $y = (\sin x)^{\sqrt{x}}$

$$\ln y = \sqrt{x}\cdot\ln(\sin x)$$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\cdot\ln(\sin x) + \sqrt{2}\cdot\frac{1}{\sin x}\cdot\cos x$$

$$y' = (\sin x)^{\sqrt{x}}\left(\frac{\ln\sin x}{2\sqrt{x}} + \frac{\sqrt{2}\cos x}{\sin x}\right)$$

6[10 pts]. Find the limits.

(a) $\lim_{x\to 0^+}(\frac{e^x}{x} - \frac{1}{x})$

$$= \lim_{x\to 0^+}\frac{e^x - 1}{x} \underset{L'H}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0^+}\frac{e^x}{1} = \frac{e^0}{1} = 1$$

(b) $\lim_{x\to\infty}(\frac{x}{x+1})^x$ $= \lim_{x\to\infty} e^{\ln\left(\frac{x}{x+1}\right)^x} = e^{\lim_{x\to\infty}\ln\left(\frac{x}{x+1}\right)^x} = e^{-1}$

$$\lim_{x\to\infty}\ln\left(\frac{x}{x+1}\right)^x = \lim_{x\to\infty} x\cdot\ln\frac{x}{x+1} = \lim_{x\to\infty}\frac{\ln\frac{x}{x+1}}{1/x}$$

$$\underset{L'H}{\overset{\frac{0}{0}}{=}} \lim_{x\to\infty}\frac{\frac{1}{x} - \frac{1}{x+1}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{\frac{1}{(x+1)\cdot x}}{-\frac{1}{x^2}} = -\lim_{x\to\infty}\frac{x^2}{(x+1)\cdot x} = -1$$