Name (Print Clearly): ________________ PID: ________________

MTH133 Section 64, Quiz 5

Nov.13, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **15** minutes.

1[3pts each]. Find the sum of the following series.

(a) $\sum_{n=1}^{\infty}(\frac{2^{n+1}}{5^n})$

$a = \frac{2^2}{5^1} = \frac{4}{5}$, $r = \frac{2}{5}$

$= \frac{a}{1-r}$

$= \frac{4/5}{1-2/5} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

(b) $\sum_{n=0}^{\infty}(-1)^n(x+1)^n$ (as a function of x)

$a = (-1)^0 \cdot (1+x)^0 = 1$. $r = -(x+1)$

$= \frac{1}{1+(x+1)} = \frac{1}{x+2}$

2 [3pts each]. Testing for convergence (show the details of your work).

(a) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}(\sqrt{n}+1)}$

$\frac{1}{\sqrt{n}(\sqrt{n}+1)} \longrightarrow \frac{1}{\sqrt{n}\cdot\sqrt{n}} = \frac{1}{n}$.

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{\sqrt{n}(\sqrt{n}+1)}} = 1$ and $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges

by lim comparison test, $\sum \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ diverges.

(b) $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$

$\lim_{n\to\infty} \frac{2n}{3n-1} = \frac{2}{3}$ does not approach 0.

By the nth term test, it diverges.

(c) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!} = \sum a_n$

$$\lim \frac{a_{n+1}}{a_n} = \lim \frac{\frac{(n+2)(n+3)}{(n+1)!}}{\frac{(n+1)(n+2)}{n!}} = \lim \frac{(n+\cancel{2})(n+3)}{(n+1)\cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{(n+1)(n+\cancel{2})}$$

$$= \lim_{n\to\infty} \frac{n+3}{(n+1)^2} = 0 < 1$$

by ratio test, it converges.

(d) [bonus 2pts] $\sum_{n=1}^{\infty} \frac{1}{n\ln(n+1)}$ $\qquad \frac{1}{n\ln(n+1)} \to \frac{1}{n\ln n}$

$\lim_{n\to\infty} \frac{\frac{1}{n\ln(n+1)}}{\frac{1}{n\ln n}} = 1$ and $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges

so $\sum_{n=1}^{\infty} \frac{1}{n\ln(n+1)} = \underbrace{\frac{1}{1\cdot\ln 2}}_{\text{finite}} + \underbrace{\sum_{n=2} \frac{1}{n\ln(n+1)}}_{\text{diverges}}$ diverges.

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ called log p-series. $\begin{cases} \text{converges}, & p>1 \\ \text{diverges}, & p\le 1. \end{cases}$