Name (Print Clearly): ________________ PID: ________________

MTH133 Section 64, Quiz 3

Oct.21, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **15** minutes.

1. Evaluate the following integrals.

(a) [4 pts] $\int \frac{dx}{(x+1)(x-1)^2} = \frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x-1| - \frac{1}{2}(x-1)^{-1} + C$

$$\frac{1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$A = \left.\frac{1}{(x-1)^2}\right|_{x=-1} = \frac{1}{4}, \quad C = \left.\frac{1}{x+1}\right|_{x=1} = \frac{1}{2}.$$

(remove the factor from $\frac{1}{(x+1)(x-1)^2}$)

multiply by x and $x \to \infty$, we have

$$0 = A + B + 0 \Rightarrow B = -\frac{1}{4}$$

(b) [4 pts] $\int \frac{y^4+y^2-1}{y^3+y}\,dy$

$$\begin{array}{r|l} & y \\ \hline y^3+y & y^4+y^2-1 \\ & y^4+y^2 \\ \hline & -1 \end{array}$$

$$= \int \left(y - \frac{1}{y^3+y}\right) dy$$

$$= \frac{1}{2}y^2 - \left[\int \left(\frac{1}{y} - \frac{y}{y^2+1}\right) dy\right]$$

$$= \frac{1}{2}y^2 - \ln|y| + \frac{1}{2}\ln|y^2+1| + C.$$

$$\frac{1}{y(y^2+1)} = \frac{A}{y} + \frac{By+C}{y^2+1}$$

$$A = \left.\frac{1}{y^2+1}\right|_{y=0} = 1;$$ multiply by y and $y \to \infty$

$$\Rightarrow 0 = A + \lim_{y\to\infty} \frac{By^2+Cy}{y^2+1} = A + B$$

So $B = -1$; Subs $y = 1$ we have

$$\frac{1}{1\cdot 2} = \frac{1}{1} + \frac{-1+C}{2} \Rightarrow C = 0.$$

$$\frac{A(y^2+1) + By^2 + Cy}{y\cdot(y^2+1)} = \frac{1}{y\cdot(y^2+1)}$$

$$\Rightarrow \begin{cases} A + B = 0 \\ C = 0 \\ A = 1 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = -1 \\ C = 0 \end{cases}$$

this method might be easier for you.

2. Evaluate the following definite integrals

(a) [4 pts] $\int_0^{\pi} \sin^4 x dx$

$$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)^2 dx = \int_0^{\pi} \frac{1-2\cos 2x+\cos^2 2x}{4} dx$$

$$= \frac{1}{4}\left[x - \sin 2x\right]_0^{\pi} + \frac{1}{4}\int_0^{\pi} \frac{1+\cos 4x}{2} dx$$

$$= \frac{\pi}{4} + \frac{1}{8}\left[x + \frac{1}{4}\sin 4x\right]_0^{\pi} = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$$

(b) [3 pts] $\int_0^5 \sqrt{25-t^2}\, dt$

let $t = 5\cdot\sin\theta$, $dt = 5\cos\theta \cdot d\theta$

$\theta = \sin^{-1}\frac{t}{5}$, when $t=0$, $\theta = 0$

when $t=5$, $\theta = \frac{\pi}{2}$

$$= \int_0^{\pi/2} \sqrt{25-25\sin^2\theta}\cdot 5\cos\theta d\theta$$

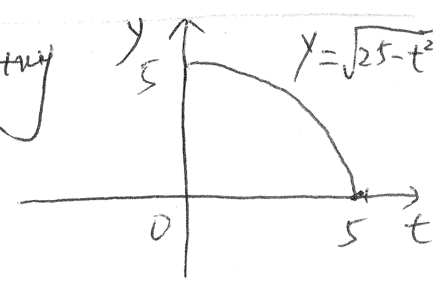
$$= \int_0^{\pi/2} 5\cdot\cos\theta\cdot 5\cdot\cos\theta\, d\theta$$

$$= 25\int_0^{\pi/2} \frac{1+\cos 2\theta}{2} d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$$

$$= \frac{25}{2}\cdot\frac{\pi}{2} = \frac{25\pi}{4}$$

By geometry



$$\int_0^5 \sqrt{25-t^2}\,dt = \text{the area}$$

$$= \frac{1}{4}\cdot\pi r^2 = \frac{\pi}{4}\cdot 25$$