Name (Print Clearly): ________________ PID: ________________

MTH133 Section 64, Quiz 2

Oct.2, 2009 Instructor: Dr. W. Wu

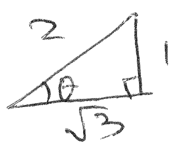
Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **15** minutes.

1 [2 pts]. Find the value of $\tan(\sin^{-1}(-1/2))$. (Hint: $\sin^{-1}(-x) = -\sin^{-1}(x)$)

Solution 1: $\sin^{-1}(-\frac{1}{2}) = -\sin^{-1}(\frac{1}{2}) = -\frac{\pi}{6}$,

$\tan(-\frac{\pi}{6}) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

Solution 2: let $\theta = \sin^{-1}\frac{1}{2}$ $\quad \tan(-\theta) = -\tan\theta = -\frac{1}{\sqrt{3}}$



2 [3pts]. Find the derivative of $y = \tan^{-1}(\ln x)$.

$$\frac{dy}{dx} = \frac{1}{1+(\ln x)^2} \cdot \frac{d}{dx}\ln x = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

3. Evaluate the integrals.

(a) [3 pts] $\int_0^1 \frac{4}{\sqrt{4-s^2}}\,ds$ $= 4\int_0^1 \frac{ds}{\sqrt{2^2-s^2}} = 4\cdot\sin^{-1}(\frac{s}{2})\Big|_0^1$

$= 4\cdot\sin^{-1}(\frac{1}{2}) - 0 = 4\cdot\frac{\pi}{6} = \frac{2\pi}{3}$

(b) [2 pts] $\int \frac{dx}{2+(x-1)^2}$ $a=\sqrt{2}$, let $u = x-1$, $du = dx$

$$= \int \frac{du}{2+u^2} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$$

(c) [2 pts] $\int \operatorname{sec}h^2(x-\frac{1}{2})dx$ let $u = x - \frac{1}{2}$, $du = dx$

$$= \int \operatorname{sech}^2 u \, du = \tanh u + C = \tanh(x-\tfrac{1}{2}) + C$$

(d) [3 pts] $\int_{-\ln 4}^{-\ln 2} 2e^x \cosh x dx$ $\cosh x = \frac{e^x + e^{-x}}{2}$

$$= \int_{-\ln 4}^{-\ln 2} 2 \cdot e^x \frac{e^x + e^{-x}}{2} dx = \int_{-\ln 4}^{-\ln 2} (e^{2x} + 1) dx$$

$$= \left[\frac{1}{2}e^{2x} + x\right]_{-\ln 4}^{-\ln 2} = \left(\frac{1}{2} \cdot e^{-2\ln 2} + (-\ln 2)\right) - \left(\frac{1}{2} e^{-2\ln 4} - \ln 4\right)$$

$$= \frac{1}{2} \cdot \frac{1}{4} - \ln 2 - \frac{1}{2} \cdot \frac{1}{16} + 2\ln 2 = \frac{3}{32} + \ln 2$$

Bonus Question [2pts]

$$\int \frac{x}{x^2 - 2x + 5} dx = \int \frac{x-1+1}{x^2-2x+5} dx = \int \frac{x-1}{x^2-2x+5} dx + \int \frac{dx}{(x-1)^2+4}$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x+5} dx + \frac{1}{2} \tan^{-1}\left(\frac{x-1}{2}\right)$$

$\left(\int \frac{f'}{f} dx = \ln|f| + C \right)$

$$= \frac{1}{2} \ln|x^2 - 2x + 5| + \frac{1}{2} \tan^{-1}\left(\frac{x-1}{2}\right) + C$$

$x^2 - 2x + 5 = (x-1)^2 + 4 > 0$

$$= \frac{1}{2} \ln(x^2 - 2x + 5) + \frac{1}{2} \tan^{-1}\left(\frac{x-1}{2}\right) + C$$