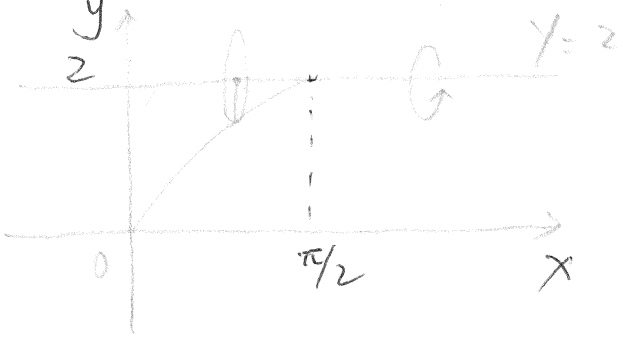
Name (Print Clearly): _____ Student Number: _____

MTH133 Section 64, Quiz 1

Sept.11, 2009 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **15** minutes.

1 [5 pts]. Find the volume of the solid generated by revolving the given region about the line y = 2. The region is in the first quadrant bounded above by the line y = 2, and below by the curve $y = 2\sin(x), 0 \le x \le \pi/2$.



$y = 2 - 2\sin x$

$$V = \pi \int_0^{\pi/2} (2 - 2\sin x)^2 \, dx$$

$$= 4\pi \int_0^{\pi/2} (1 - 2\sin x + \sin^2 x) \, dx$$

$$= 4\pi \int_0^{\pi/2} \left(1 - 2\sin x + \frac{1 - \cos 2x}{2}\right) dx$$

$$= 4\pi \int_0^{\pi/2} \left(\frac{3}{2} - 2\sin x - \frac{1}{2}\cos 2x\right) dx$$

$$= 4\pi \left[\frac{3}{2}x + 2\cos x - \frac{1}{4}\sin 2x\right]_0^{\pi/2}$$

$$= 4\pi \left[\left(\frac{3}{2}\cdot\frac{\pi}{2} + 2\cdot\cos\frac{\pi}{2} - \frac{1}{4}\sin\pi\right) - (0 + 2\cos 0 - 0)\right]$$

$$= 3\pi^2 - 8\pi$$

OVER =>

2 [5pts]. Find the length of the curve: $x = \cos t$, $y = t + \sin t$, $0 \le t \le \pi$.

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\pi} \sqrt{(-\sin t)^2 + (1+\cos t)^2}\, dt$$

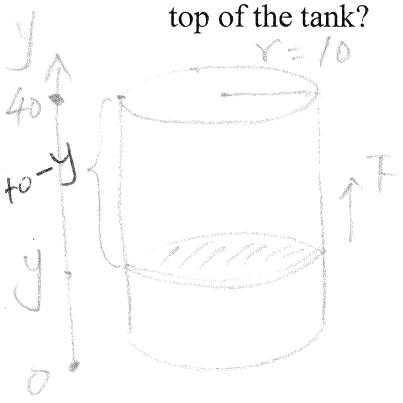
$$= \int_0^{\pi} \sqrt{\sin^2 t + 1 + 2\cos t + \cos^2 t}\, dt$$

$$= \int_0^{\pi} \sqrt{2 + 2\cos t}\, dt = \int_0^{\pi} \sqrt{4 \cdot \frac{1+\cos t}{2}}\, dt$$

$$= 2\int_0^{\pi} \sqrt{\cos^2 \frac{t}{2}}\, dt = 2\int_0^{\pi} \cos\frac{t}{2}\, dt$$

$$= 2 \cdot 2\sin\frac{t}{2}\Big|_0^{\pi} = 4 \cdot \sin\frac{\pi}{2} - 0 = 4$$

3 [5 pts]. A vertical right circular cylindrical tank measures 40 ft high and 20 ft in diameter. It is full of gasoline weighting 50 lb/ ft^3. How much work does it take to pump the gasoline to the top of the tank?



Consider the thin slab at height = y, $0 \le y \le 40$

the thickness = Δy.

$\Delta V = \pi \cdot r^2 \cdot \Delta y = 100\pi \cdot \Delta y$

$F = \text{Weight} = \Delta V \cdot 50 = 5000\pi \cdot \Delta y$.

$\Delta W = F \cdot d = 5000\pi \cdot \Delta y \cdot (40 - y)$

as the thickness $\to 0$. $\Delta y \to dy$

$$W = \int_0^{40} 5000\pi(40-y)\, dy = 5000\pi \cdot \int_0^{40} (40-y)\, dy$$

$$= 5000\pi\left[40y - \frac{y^2}{2}\right]_0^{40} = 5000\pi \cdot 800$$

$$= 40 \times 10^5 \times \pi \text{ lb} \cdot \text{ft}$$