Name (Print Clearly): Student Number:

MTH132 Section 5 & 18, Test 4

Dec 1, 2008 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [5 pts each]. Find the derivatives.

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\cdot b'(x) - f(a(x))\cdot a'(x)$$

(a) $y = \int_0^x (\sin^2 t)dt$

$y' = \sin^2 x \cdot (x)' - \sin^2 0 \cdot 0' = \sin^2 x$

(b) $y = \int_{x^2}^0 t(1+t^2)dt$

$a(x) = x^2$, $b(x) = 0$, $\therefore b'(x) = 0$.

$y' = -x^2(1+x^4)\cdot a'(x) = -2x^3(1+x^4)$

2. [6 pts each]. Find indefinite integrals.

(a) $\int 4x\sqrt{1+2x^2}\,dx$

let $t = 1+2x^2$. So $dt = 4x\,dx$

$= \int \sqrt{t}\,dt$

$= \frac{2}{3}t^{3/2} + C = \frac{2}{3}(1+2x^2)^{3/2} + C$

(b) $\int \frac{x^2-\sqrt{x}}{x}dx$

$= \int (x - x^{-1/2}) dx = \int x\,dx - \int x^{-1/2} dx$

$= \frac{1}{2}x^2 - 2x^{1/2} + C$

3. [7 pts each]. Evaluate the definite integrals.

(a) $\int_0^2 x(1-x^3)dx$

$= \int_0^2 (x - x^4) dx = \left[\frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^2 = \left(2 - \frac{32}{5}\right) - 0 = -\frac{22}{5}$

(b) $\int_{-\pi/2}^{\pi/2} (2\sin^3 x \cdot \cos x)dx$

Solution1. $f(x) = 2\sin^3 x \cdot \cos x$

$f(-x) = 2\sin^3(-x) \cdot \cos(-x) = 2(-\sin x)^3 \cdot \cos x = -2\sin^3 x \cdot \cos x$

$\therefore$ $f(x)$ is odd. Also, $[-\pi/2, \pi/2]$ is a symmetric interval

So $\int_{-\pi/2}^{\pi/2} f(x)\,dx = 0$.

Solution 2. let $t = \sin x$, $dt = \cos x \cdot dx$, $\sin(\pi/2) = 1$, $\sin(-\pi/2) = -1$

$\therefore \int_{-1}^{1} 2t^3 \cdot dt = \frac{1}{2}t^4 \Big|_{-1}^{1} = \frac{1}{2}1^4 - \frac{1}{2}(-1)^4 = 0$.

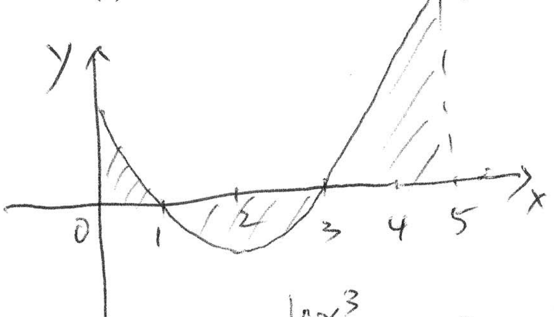
4. [12 pts]. Let $f(x) = x^2 - 4x + 3$, $0 \le x \le 5$.

(a) Compute $\int_0^5 f(x)dx$

$$= \frac{x^3}{3} - 2x^2 + 3x \Big|_0^5 = \frac{125}{3} - 2 \times 25 + 15 = \frac{125}{3} - 35$$

$$= \frac{125}{3} - \frac{105}{3} = \frac{20}{3}$$

(b) Find the total area enclosed by x-axis and the graph of $f(x)$ over $[0,5]$

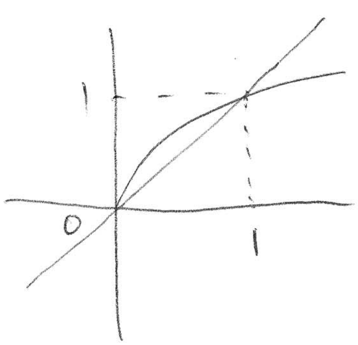


$$\text{Total Area} = \left| \int_0^1 f(x)dx \right| + \left| \int_1^3 f(x)dx \right| + \left| \int_3^5 f(x)dx \right|$$

$$= \left| \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 \right| + \left| \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3 \right| + \left| \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^5 \right|$$

$$= \left|\frac{4}{3}\right| + \left|-\frac{4}{3}\right| + \left|\frac{20}{3}\right| = \frac{28}{3}$$

5 [12 pts]. Find the area of the region enclosed by the curve $x = y^2$ and the line $y = x$.



solution 1. integration w.r.t x

$y = \sqrt{x}$, $y = x$, (upper curve: $y = \sqrt{x}$)

$$\text{Area} = \int_0^1 (\sqrt{x} - x)dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{2}x^2\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

solution 2. integration w.r.t. y

$x = y$ (right-side curve)

$x = y^2$

$$\text{Area} = \int_0^1 (y - y^2)dy = \left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

6. [10 pts] Set up a Riemann sum approximation to the integral $\int_1^3 (x^2+1)dx$ by partitioning the interval $[1,3]$ into 4 subintervals of equal length and using the left endpoint of each subinterval.

(a) Evaluate this sum.

$$\Delta X = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}.$$

$$X_k = a + k\cdot\Delta X = 1 + \frac{k}{2}$$

$C_k = X_{k-1}$ (using the left endpt)

$\therefore C_k = 1 + \frac{k-1}{2} = \frac{k+1}{2}$ (replace k by $k-1$ in $X_k = 1+\frac{k}{2}$)

$$f(C_k) = \left(\frac{k+1}{2}\right)^2 + 1$$

$$S = \sum_{k=1}^{4} f(C_k)\cdot\Delta X = \sum_{k=1}^{4}\left[\left(\frac{k+1}{2}\right)^2+1\right]\cdot\frac{1}{2}$$

$$= \frac{1}{2}\left[2 + \left(1+\frac{9}{4}\right) + (4+1) + \left(\frac{25}{4}+1\right)\right] = \frac{35}{4}$$

(b). Sketch the graph of $y = x^2 + 1$ over $[1,3]$. Sketch (shade) the rectangles associated to this Riemann Sum.