Name (Print Clearly): Student Number:

MTH132 Section 5 & 18, Test 3

Nov 7, 2008 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [4 pts each]. Find limits of each of the following functions.

(a) $\lim_{x\to 1}\frac{x^{3/5}-1}{x^{2/3}-1}$

$$\overset{L'H}{\underset{\frac{0}{0}}{=}} \lim_{x\to 1}\frac{\frac{3}{5}x^{-2/5}}{\frac{2}{3}x^{-1/3}} \overset{DSP}{=} \frac{3}{5}\cdot\frac{3}{2} = \frac{9}{10}$$

(b) $\lim_{x\to 0}\frac{\sin^2 x}{\sin(x^2)}$

$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\frac{x^2}{\sin x^2}$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\cdot\lim_{x\to 0}\frac{x^2}{\sin x^2}$$

$$= 1^2\cdot 1 = 1$$

or

$$\overset{L'H}{\underset{\frac{0}{0}}{=}} \lim_{x\to 0}\frac{2\sin x\cdot\cos x}{\cos x^2\cdot 2x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\cos x}{\cos x^2}$$

$$= 1\cdot\frac{1}{1} = 1$$

(c) $\lim_{x\to\infty}\frac{x-2x^2-3}{3x^2+x^{3/2}}$

$$= \lim_{x\to\infty}\frac{1/x-2-3/x^2}{3+x^{-1/2}}$$

$$= \frac{0-2-0}{3+0} = -\frac{2}{3}$$

or

$$\overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty}\frac{1-4x}{6x+\frac{3}{2}x^{1/2}}$$

$$\overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty}\frac{-4}{6+\frac{3}{4}x^{-1/2}}$$

$$= \frac{-4}{6+0} = -\frac{2}{3}$$

2. [4 pts each]. Find indefinite integrals.

(a) $\int (x^{-1/3} + 3x^2 - 2)dx$

$= \frac{1}{-\frac{1}{3}+1} x^{2/3} + x^3 - 2x + C = \frac{3}{2} x^{2/3} + x^3 - 2x + C$

(b) $\int \frac{1}{(r+5)^2} dr$ let $t = r+5 \Rightarrow dt = dr$

$= \int \frac{1}{t^2} dt = -t^{-1} + C = -(r+5)^{-1} + C$

(c) [4 pts bonus] $\int \frac{\sin t}{1-\sin^2 t} dt$

$= \int \frac{\sin t}{\cos^2 t} dt = \int \sec t \cdot \tan t \cdot dt = \sec t + C$

or let $u = \sin t$, $du = \cos t \cdot dt$ so $\int \frac{u}{1-u^2} \cdot \frac{du}{\cos t} = \int \frac{u\,du}{(1-u^2)^{3/2}}$

$\cos t = \sqrt{1-u^2}$

let $r = 1-u^2 \Rightarrow dr = -2u\,du$

$-\frac{1}{2}\int r^{-3/2} dr = r^{-1/2} + C$

$= (1-u^2)^{-1/2} + C$

$= (1-\sin^2 t)^{-1/2} + C$

$= \frac{1}{\cos t} + C$

3. [8 pts]. Solve initial value problem: $\frac{dy}{dx} = \cos(\pi \cdot x), y(0) = 1$.

$y = \int \cos \pi x\, dx$

$= \frac{1}{\pi} \sin \pi x + C$

$y(0) = C = 1$

$\therefore y(x) = \frac{1}{\pi} \sin \pi x + 1$

4. [10 pts]. Let $f(x) = x + \frac{1}{x}$, $[\frac{1}{2}, 2]$.

(a) Show that f(x) satisfies the hypotheses of the Mean Value Theorem over the given interval.

x is cont. and differentiable on R

$\frac{1}{x}$ is cont. and differentiable on $(0, \infty)$

So $f(x) = x + \frac{1}{x}$ is cont. and diff on $[\frac{1}{2}, 2]$

(b) Find c which satisfies the equation of the Mean Value Theorem: $\frac{f(b)-f(a)}{b-a} = f'(c)$, on $[\frac{1}{2}, 2]$.

$$\left.\begin{array}{l}\frac{f(2)-f(\frac{1}{2})}{2-\frac{1}{2}} = 0 = f'(c) \\ f'(x) = 1 - \frac{1}{x^2}\end{array}\right\} \Rightarrow 1 - \frac{1}{c^2} = 0 \Rightarrow c = 1 \in [\tfrac{1}{2}, 2]$$

5 [12 pts]. Let $f(x) = \frac{x^2}{(x-3)^2}$ (Note that $f'(x) = \frac{-6x}{(x-3)^3}$ and $f''(x) = \frac{12x+18}{(x-3)^4}$)

(a) Find the intervals where f is increasing or decreasing.

$f' = -6x = 0 \Rightarrow x = 0$

f' is undefined $\Rightarrow x - 3 = 0 \Rightarrow x = 3$

Sign of f': $-$ (0) $+$ (3) $-$

increasing: $(0, 3)$

decreasing: $(-\infty, 0)$, $(3, \infty)$

(b) Find the intervals where f is concave up or concave down.

$f'' = 12x + 18 = 0 \Rightarrow x = -\frac{3}{2}$

f'' is undefined $\Rightarrow x = 3$.

Sign of f'': $-$ ($-3/2$) $+$ (3) $+$

concave up: $(-3/2, 3)$, $(3, \infty)$

concave down: $(-\infty, -3/2)$

(c) Find all local extrema and inflection points.

local min at $x = 0$ $f(0) = 0$.

no local max ~~at $x=3$ $f(3)$~~ f is undefined at $x=3$ and $\lim_{x\to 3} f(x) = \infty$

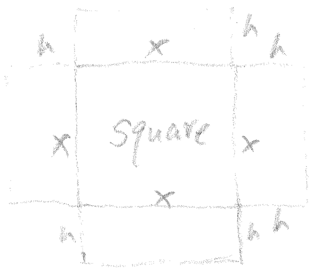
inflection point at $x = -3/2$

(d) Find the equations of all asymptotes.

vertical asymptote: $x = 3$. since $\lim_{x\to 3} f(x) = \infty$

horizontal asymptote: $y = 1$, since $\lim_{x\to\pm\infty} f(x) = 1$

6 [10 pts]. A rectangular box with volume 18 cubic meters is to be built with a **square** base and **NO** top. The material used for the bottom panel costs \$2 per square meter while the material used for the side panels costs \$1.5 per square meter. Find the minimum cost to build such a box.



given info: $V = x^2 \cdot h = 18$

target func: $C = 4 \cdot x \cdot h \cdot 1.5 + x^2 \cdot 2$

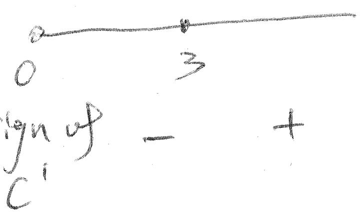
$h = \frac{18}{x^2}$ so new func $C = 6 \cdot x \cdot \frac{18}{x^2} + 2x^2$

$C = \frac{108}{x} + 2x^2 \Rightarrow C' = 4x - \frac{108}{x^2}$

dom = $(0, \infty)$

$C' = 0 \Rightarrow 4x - \frac{108}{x^2} = 0 \Rightarrow 4x^3 - 108 = 0 \Rightarrow$

$x = 3$ and $\cancel{h = \frac{18}{27} = \frac{2}{3}}$ $h = \frac{18}{9} = 2$



so C has local minimum at $x = 3$; $C = 54$.

It is also the global minimum.

7. [10 pts]. Use Newton's Method to approximate $\sqrt{15}$.

(a) Construct a function $f(x)$ such that $\sqrt{15}$ is a root of $f(x) = 0$

let $x = \sqrt{15}$, $x^2 = 15 \Rightarrow x^2 - 15 = 0$

so $f(x) = x^2 - 15$, then $\sqrt{15}$ is a root of $f(x) = 0$.

(b) Find an initial guess x_0 of the solution then use Newton's method to construct x_1 and x_2.

let $x_0 = 4$. $f'(x) = 2x$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4 - \frac{1}{8} = \frac{31}{8}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{31}{8} - \frac{(\frac{31}{8})^2 - 15}{2 \cdot \frac{31}{8}} = \frac{1921}{496}$$