Name (Print Clearly): Student Number:

MTH132 Section 5 & 18, Test 2

Oct 10, 2008 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [5 pts each]. Find dy/dx of each of the following functions. **Do Not Simplify**

(a) $y = \sqrt{3-x}$

$y = (3-x)^{1/2} \Rightarrow y' = \frac{1}{2}(3-x)^{-1/2} \cdot \frac{d}{dx}(3-x) = -\frac{1}{2}(3-x)^{-1/2}$

(b) $y = (4x+3)^4$

$y' = 4(4x+3)^3 \cdot \frac{d}{dx}(4x+3) = 16 \cdot (4x+3)^3$

(c) $y = (\csc x + \cot x)^{-1}$

$y' = -1(\csc x + \cot x)^{-2} \cdot \frac{d}{dx}(\csc x + \cot x)$

$= -(\csc x + \cot x)^{-2} \cdot (-\csc x \cdot \cot x - \csc^2 x)$

(d) $y = \dfrac{1}{x}\sin^{-5} x - \dfrac{x}{3}\cos^3 x$

$y' = -x^{-2}\sin^{-5} x + \frac{1}{x}(-5)\sin^{-6} x \cdot \cos x - \left(\frac{1}{3}\cos^3 x + \frac{x}{3} \cdot 3 \cdot \cos^2 x \cdot (-\sin x)\right)$

(e) $y = \dfrac{x^2 + x + 1}{\sqrt{x^2 + 1}}$

$g = (x^2+1)^{1/2} \Rightarrow g' = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$y' = \dfrac{(2x+1)\sqrt{x^2+1} - (x^2+x+1) \cdot x(x^2+1)^{-1/2}}{x^2+1}$

(f) $x^3 + 4xy - 3y^{4/3} = 2x$ (use implicit differentiation)

use $\frac{d}{dx}$: $3x^2 + 4y + 4xy' - 3 \cdot \frac{4}{3} y^{1/3} \cdot y' = 2$

$(4x - 4y^{1/3})y' = 2 - 4y - 3x^2$

$y' = \dfrac{2 - 4y - 3x^2}{4x - 4y^{1/3}}$

2 [5 pts each]. Let $f(x) = \dfrac{1}{2x+1}$.

(a) Find the derivative using the definition.

$f'(x) = \lim\limits_{z \to x} \dfrac{f(z) - f(x)}{z - x} = \lim\limits_{z \to x} \dfrac{\frac{1}{2z+1} - \frac{1}{2x+1}}{z - x} = \lim\limits_{z \to x} \dfrac{(2x+1) - (2z+1)}{(z-x)(2z+1)(2x+1)}$

$= \lim\limits_{x \to z} \dfrac{-2(z-x)}{(z-x)(2x+1)(2z+1)} = \lim\limits_{z \to x} \dfrac{-2}{(2x+1)(2z+1)} = \dfrac{-2}{(2x+1)^2}$

(b) Find the derivative using differentiation rules.

$f(x) = (2x+1)^{-1} \Rightarrow f' = -(2x+1)^{-2} \cdot 2.$

(1, 1)

3 [10 pts]. Show that $(1,-1)$ lies on the curve $x^3y^3 + y^2 = x + y$. Then find an equation for the tangent line to the curve at this point.

Subs $x=1,\ y=-1$ into $x^3y^3+y^2=x+y$

$1^3(-1)^3+(-1)^2 = 1-1 \Rightarrow -1+1 = 1-1 \Rightarrow 0=0$

use $\frac{d}{dx}$: $2yy'+3x^2y^3+3x^3y^2\cdot y' = 1+y'$

$(2y+3x^3y^2-1)y' = 1-3x^2y^3$

subs $x=1,\ y=-1$, $(-2+3-1)\,y' = 1+3 = 4$

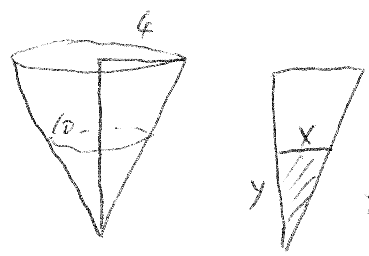
y' is undefined at $(1,-1)$.

$y'\to\infty$, it is a vertical tangent

so $x=1$ is the tangent line.

4 [10 pts]. Water drains from a conical tank at the rate of $5m^3/\text{min}$. The tank stands point down and has a height $10m$ and a base radius of $4m$. Suppose the water is $6m$ deep at this moment.

(a) What is the relation between radius of the surface and the depth of the water?



use similar Δs

$\frac{4}{10} = \frac{x}{y}$

x: radius of the surface of water

y: height - - - - - -

(b) What is the relation between the volume and depth of the water?

V: Volume of the water, so $V=\frac{\pi}{3}x^2\cdot y$

$\because x=\frac{2}{5}y \quad \therefore V=\frac{\pi}{3}\left(\frac{2}{5}y\right)^2\cdot y = \frac{4\pi}{75}y^3$

(c) How fast is the water level dropping at this moment?

take $\frac{d}{dt}$: $V(t)=\frac{4\pi}{75}y^3(t)$, when $y=6$

$\frac{dV}{dt} = \frac{4\pi}{25}\cdot y^2\cdot\frac{dy}{dt}$

$-5 = \frac{4\pi}{25}\cdot 36\cdot\frac{dy}{dt}$

so $\frac{dy}{dt} = -\frac{125}{144\pi}$

5. [10 pts]. Let $f(x)=\sqrt{1+x}+\sin x-0.5$

$L(x)=f(a)+f'(a)(x-a)$ at $x=a$.

(a) Find the linearization of $f(x)$ at $x=0$.

$f(0)=\sqrt{1+0}+\sin 0-0.5=1-0.5=0.5$

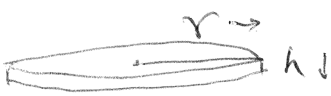
$f'(0)=(\sqrt{1+x}+\sin x-0.5)'\Big|_{x=0}=\frac{1}{2}(1+x)^{-1/2}+\cos x\Big|_{x=0}=\frac{1}{2}+\cos 0=1.5$

$\therefore\ L(x)=f(0)+f'(0)(x-0)=0.5+1.5x$

(b) Estimate $f(0.1)$ using the result of (a).

$f(0.1)\approx L(0.1)=0.5+1.5\times 0.1=0.65$

6. [4 pts bonus]. When a tanker collides with a boat, $100\pi\ m^3$ of oil is spilled in the ocean. The resulting oil slick forms a right circular cylinder on the surface of the water. If the thickness of the slick is decreasing at a rate of $0.001 m/\sec$, how fast is the radius increasing when the slick is $0.01m$ thick?



r: the radius of the cylinder, V: Volume of ...

h: " height of " " "

$\frac{dh}{dt}=-0.001$, $h=0.01$, $V=100\pi$, $\frac{dr}{dt}=?$

$V=\pi r^2\cdot h$, V is a const, r and h are functions of t

$100\pi=\pi r^2\times 0.01 \Rightarrow r=100$

$V=\pi r^2(t)\cdot h(t)$

$\frac{dV}{dt}=\pi\left(2r\cdot\frac{dr}{dt}\cdot h+r^2\cdot\frac{dh}{dt}\right)=0$

$\Rightarrow 2h\frac{dr}{dt}+r\cdot\frac{dh}{dt}=0$

$\Rightarrow 0.02\frac{dr}{dt}+100\cdot(-0.001)=0$

$\Rightarrow 0.02\frac{dr}{dt}=0.1 \Rightarrow \frac{dr}{dt}=5$