Name (Print Clearly): Student Number:

MTH132 Section 5 & 18, Test 1

Sept 15, 2008 Instructor: Dr. W. Wu

Instructions: Answer the following questions in the space provided. There is more than adequate space provided to answer each question. The total time allowed for this quiz is **50** minutes.

1 [4 pts each]. Find the following limits.

(a) $\lim_{x \to 0} \frac{\sqrt{x^2+2}-x}{x^2+x-1}$

$$= \frac{\sqrt{0+2}-0}{0+0-1} = -\sqrt{2}$$

(b) $\lim_{x \to 3} \frac{x^2-x-6}{x^2-3x}$

$$= \lim_{x \to 3} \frac{(x-3)(x+2)}{x(x-3)} = \lim_{x \to 3} \frac{x+2}{x} = \frac{5}{3}$$

(c) $\lim_{x \to 0} x\cot(2x)$

$$= \lim_{x \to 0} x \cdot \frac{\cos 2x}{\sin 2x} = \lim_{x \to 0} \frac{2x}{\sin 2x} \cdot \frac{\cos 2x}{2} = \lim_{x \to 0} \frac{2x}{\sin 2x} \cdot \lim_{x \to 0} \frac{\cos 2x}{2}$$

$$= 1 \cdot \frac{\cos 0}{2} = \frac{1}{2}$$

(d) $\lim_{x \to \infty} \frac{x^2-x-6}{x^2-3x}$

$$= \lim_{x \to \infty} \frac{1-1/x-6/x^2}{1-3/x} = \frac{1-0-0}{1-0} = 1$$

(e) $\lim_{x \to 3^-} \frac{x^2-6}{x^2-3x}$

as $x \to 3^-$, $x^2-6 \to 3$ and $x^2-3x \to 0^-$

so $\lim_{x \to 3^-} \frac{x^2-6}{x^2-3x} = -\infty$

2 [5 pts each]. If $f(x) = 1/\sqrt{x}$, please evaluate the limit of the form $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$,

(a) when $x = 4$.

$f(x) = \frac{1}{\sqrt{x}} \Rightarrow f(x+h) = \frac{1}{\sqrt{x+h}}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ when $x = 4$

$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{2}}{h} = \lim_{h \to 0} \frac{2 - \sqrt{4+h}}{2h\sqrt{4+h}} = \lim_{h \to 0} \frac{-h}{2h\sqrt{4+h}(2+\sqrt{4+h})}$

$= \lim_{h \to 0} \frac{-1}{2\sqrt{4+h}(2+\sqrt{4+h})} = \frac{-1}{2\sqrt{4+0}(2+\sqrt{4+0})} = \frac{-1}{2 \cdot 2 \cdot (2+2)} = -\frac{1}{16}$

(b) when $x = x_0$.

$\lim_{h \to 0} \frac{\frac{1}{\sqrt{x_0+h}} - \frac{1}{\sqrt{x_0}}}{h} = \lim_{h \to 0} \frac{\sqrt{x_0} - \sqrt{x_0+h}}{h\sqrt{x_0}\sqrt{x_0+h}} = \lim_{h \to 0} \frac{-h}{h\sqrt{x_0}\sqrt{x_0+h}(\sqrt{x_0}+\sqrt{x_0+h})}$

$= \lim_{h \to 0} \frac{-1}{\sqrt{x_0}\sqrt{x_0+h}(\sqrt{x_0}+\sqrt{x_0+h})} = \frac{-1}{\sqrt{x_0}\sqrt{x_0+0}(\sqrt{x_0}+\sqrt{x_0+0})} = -\frac{1}{2x_0\sqrt{x_0}}$

3 [10 pts]. Please use the intermediate value theorem to show that the equation $\cos(2x) - 2\sin x = 0$ has a solution.

① let $y = \cos 2x - 2\sin x$, it is a continuous function on $\mathbb{R}$ since $\cos 2x$ and $\sin x$ are continuous.

②. let $[a, b] = [0, \frac{\pi}{2}]$.

$y(0) = \cos 0 - 2\sin 0 = 1$

$y(\frac{\pi}{2}) = \cos \pi - 2\sin\frac{\pi}{2} = -1 - 2 = -3$

$\therefore \quad y(0) > 0 > y(\frac{\pi}{2})$

③. by the intermediate value theorem there exists c in $[0, \frac{\pi}{2}]$ such that $y(c) = 0$. so c is a solution of this equation.

4 [5 pts each]. Suppose $y = \dfrac{x^2 - 3}{x - 2}$.

$$\begin{array}{r} x + 2 \\ x-2 \,\overline{)\, x^2 - 3} \\ \underline{x^2 - 2x} \\ 2x - 3 \\ \underline{2x - 4} \\ 1 \end{array}$$

(a) Find the asymptotes of this function.

$$\frac{x^2-3}{x-2} = x+2+\frac{1}{x-2}$$

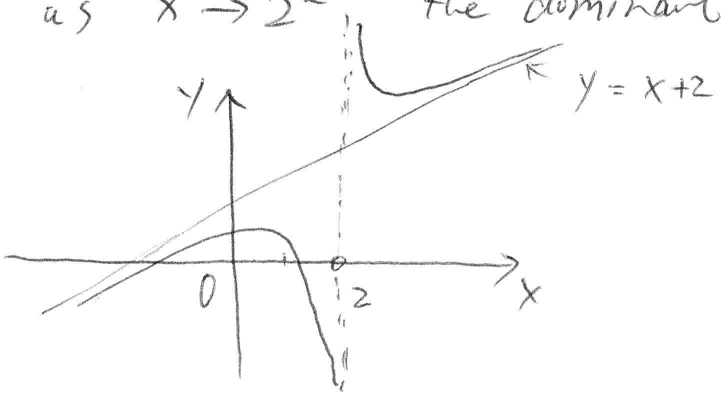
$\therefore$ $y = x+2$ is an oblique asymptote

$x = 2$ is a vertical asymptote.

(b) Find the dominant terms and sketch the graph of this function.

as $x \to \pm\infty$. $y \to x+2$, dominant term is $x+2$.

as $x \to 2^{\pm}$, the dominant term is $\frac{1}{x-2}$



as $x \to 2^{+}$, $y \to +\infty$

as $x \to 2^{-}$ $y \to -\infty$

as $x \to \infty$, $y \to x+2$ and $y > x+2$

as $x \to -\infty$ $y \to x+2$ and $y < x+2$

5 [5 pts each]. At which points are each of the following functions continuous?

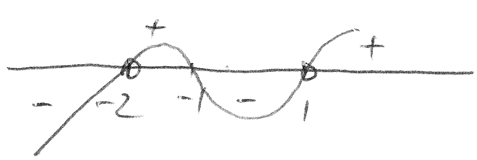
(a) $y = \sqrt{\dfrac{x+1}{x^2 + x - 2}}$

① domain of this function

$$\frac{x+1}{x^2+x-2} \geq 0 \quad \text{and} \quad x^2+x-2 \neq 0 \Rightarrow x \neq 1, -2$$

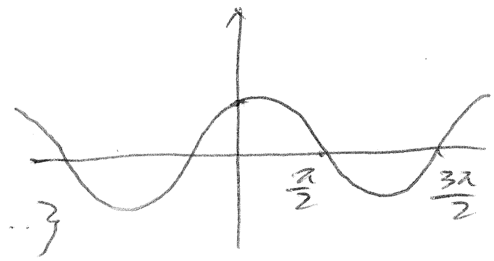
$$\therefore \frac{x+1}{x^2+x-2} \cdot (x^2+x-2)^2 \geq 0 \Rightarrow (x+1)(x-1)(x+2) \geq 0$$

$\therefore$ dom $= (-2, -1] \cup (1, \infty)$



②. it is cont. on its dom.

(b) $y = \dfrac{x+2}{\cos x}$



① it implies $\cos x \neq 0$, so
$x \neq \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z} = \{0, \pm 1, \pm 2 \cdots\}$

② it is cont. when $x \neq \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

6 [10 pts]. Use the Sandwich Theorem to find $\lim\limits_{x\to\infty} \dfrac{x\sin(x^2-3)}{x^2-3}$.

①. $-1 \leq \sin(x^2-3) \leq 1$, (1) , $\because x \to \infty \quad \therefore \dfrac{x}{x^2-3} > 0$

② $-\dfrac{x}{x^2-3} \leq \dfrac{x\sin(x^2-3)}{x^2-3} \leq \dfrac{x\sin(x^2-3)}{x^2-3}$, multiply (1) by $\dfrac{x}{x^2-3}$

③ $\lim\limits_{x\to\infty} \dfrac{-x}{x^2-3} = \lim\limits_{x\to\infty} \dfrac{-1/x}{1-3/x^2} = \dfrac{-0}{1-0} = 0$

$\lim\limits_{x\to\infty} \dfrac{x}{x^2-3} = \lim\limits_{x\to\infty} \dfrac{1/x}{1-3/x^2} = \dfrac{0}{1-0} = 0$.

④. by the Sandwich Theorem
$\lim\limits_{x\to\infty} \dfrac{x\sin(x^2-3)}{x^2-3} = 0$.