11.10 (a) $\forall n \in \mathbb{N}$, there are infinitely many $\frac{1}{n}$s $\Rightarrow \frac{1}{n} \in S$.

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \cdots\}$ is a subsequence $\Rightarrow 0 \in S$.

In all, $S = \{0\} \cup \{\frac{1}{n}\}_{n \in \mathbb{N}}$

(b) $\limsup s_n = \sup S = 1$. $\liminf s_n = \inf S = 0$.

14.6 (a) $\{b_n\}$ is bounded $\Rightarrow \exists M > 0$, s.t. $|b_n| \leq M$, $\forall n \in \mathbb{N}$.

$\forall \epsilon > 0$, consider $\epsilon_1 = \frac{\epsilon}{M} > 0$. Since $\sum |a_n|$ converges, it is Cauchy

$\Rightarrow \exists N > 0$, $\forall n \geq m > N$, $\left| \sum_{k=m+1}^{n} |a_k| \right| < \epsilon_1 = \frac{\epsilon}{M} \Rightarrow \sum_{k=m+1}^{n} |a_k| < \frac{\epsilon}{M}$

$\Rightarrow \left| \sum_{k=m+1}^{n} a_k b_k \right| \leq \sum_{k=m+1}^{n} |a_k b_k| \leq M \sum_{k=m+1}^{n} |a_k| < M \cdot \frac{\epsilon}{M} = \epsilon$

$\Rightarrow \sum a_n b_n$ is Cauchy hence convergent.

15.4 (a) $\lim_{n \to \infty} \frac{\log n}{\sqrt{n}} = 0 \Rightarrow$ for $\epsilon = 1 > 0$, $\exists N > 0$, $\forall n > N$ $\frac{\log n}{\sqrt{n}} < 1$

$\Rightarrow \frac{1}{\sqrt{n}} < \frac{1}{\log n} \Rightarrow \frac{1}{\sqrt{n}} \cdot \frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}} \cdot \frac{1}{\log n}$, i.e., $\frac{1}{n} < \frac{1}{\sqrt{n} \cdot \log n}$

$\sum_{n=N+1}^{\infty} \frac{1}{n}$ diverges $\Rightarrow \sum_{n=N+1}^{\infty} \frac{1}{\sqrt{n} \cdot \log n}$ diverges $\Rightarrow \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \cdot \log n} = \sum_{n=2}^{N} \frac{1}{\sqrt{n} \log n} + \sum_{n=N+1}^{\infty} \frac{1}{\sqrt{n} \log n}$ diverges.

(b) $n \geq 3$, $\log n > 1 \Rightarrow \frac{\log n}{n} > \frac{1}{n}$, $\forall n \geq 3$.

$\sum_{n=3}^{\infty} \frac{1}{n}$ diverges $\Rightarrow \sum_{n=3}^{\infty} \frac{\log n}{n}$ diverges $\Rightarrow \sum_{n=2}^{\infty} \frac{\log n}{n} = \frac{\log 2}{2} + \sum_{n=3}^{\infty} \frac{\log n}{n}$ diverge

(d) $\lim_{n \to \infty} \frac{\log n}{\sqrt{n}} = 0 \Rightarrow \exists N > 0$, $\forall n > N$, $\frac{\log n}{\sqrt{n}} < 1 \Rightarrow \frac{\log n}{n^2} < \frac{\sqrt{n}}{n^2} = \frac{1}{n^{\frac{3}{2}}}$

$\sum_{n=N+1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ converges $\Rightarrow \sum_{n=N+1}^{\infty} \frac{\log n}{n^2}$ converges $\Rightarrow \sum_{n=2}^{\infty} \frac{\log n}{n^2} = \sum_{n=2}^{N} \frac{\log n}{n^2} + \sum_{n=N+1}^{\infty} \frac{\log n}{n^2}$ Converges.