FS25 MTH994: Mathematics-Assisted AI and AI-inspired Mathematics

Instructor: Guowei Wei – wei@math.msu.edu

Course Description:

Artificial intelligence (AI), including machine learning (ML) and deep learning (DL), has fundamentally changed the landscape of science, engineering, and technology over the past decade. While linear algebra, calculus, statistics, and probability underpin AI, modern mathematical fields such as geometry, topology, nonlinear algebra, analysis, and combinatorics are a driving force in data science and hold the future of AI. The overall goal of this course is to explore mathematics-assisted AI and AI-inspired mathematics, emphasizing methodologies that connect mathematical theories, concepts, and frameworks to AI and data science.

We will review how algebraic topology, differential topology, and geometric topology have been adapted for data science and AI over the past decade. Since the field encompasses so many topics, I will only emphasize recent advances in topological data analysis and topological deep learning. The latter was first introduced by Cang and Wei in 2017. Additionally, we will introduce several topics in differential geometry, including de Rham cohomology, Hodge decomposition, Gauss, mean, and Ricci curvatures, and Grassmannian manifolds, in the context of data science and geometric learning. We will also discuss the application of optimal transport techniques, such as Wasserstein and Gromov-Wasserstein distances, to data science and AI. Furthermore, we will discuss recently established connections between AI and commutative algebra, algebraic combinatorics, and polynomial rings, highlighting their promising potential.

If time permits, we will briefly discuss the current challenges in applying Chern classes, Atiyah–Singer index theory, quantum topology, and Heegaard Floer homology to data science and AI.

Prerequisites: A student must have done with his/her math qualifying sequences.

Text: There is no required textbook for this course.