3.9 Problems

Standard Problems

Example 1. Find the most general antiderivative of the function:

(a) $f(x) = \sqrt{x^3} + \sqrt[3]{x^2}$ $= x^{3/2} + x^{2/3}$

$F(x) = \frac{2}{5} x^{5/2} + \frac{3}{5} x^{5/3} + C$

(b) $g(x) = 8x - 3\sec^2 x$

$G(x) = 8 \cdot \frac{1}{2} x^2 - 3\tan x + C$

(c) $h(t) = \dfrac{3 - t + t^2}{\sqrt{t}}$ $= 3t^{-\frac{1}{2}} - t^{\frac{1}{2}} + t^{3/2}$

$H(t) = 6t^{\frac{1}{2}} - \frac{2}{3} t^{3/2} + \frac{2}{5} t^{5/2} + C$

Example 2. Find the function f that satisfies: $f''(t) = \sin t + \cos t$, $f'(0) = 4$, $f(0) = 3$.

$f'(t) = -\cos t + \sin t + C$ (1) antiderivative

$f'(0) = -1 + C$, $f'(0) = 4$ $\Rightarrow$ $C = 5$ (2)

plug (2) into (1): $f'(t) = -\cos t + \sin t + 5$ antiderivative

$f(t) = -\sin t - \cos t + 5t + C$

$f(0) = -1 + C = 3 \Rightarrow C = 4$ plug in

Example 3. Solve the initial value problem: $f''(x) = 3/\sqrt{x}$, $f'(4) = 7$, $f(4) = 20$.

$= 3x^{-\frac{1}{2}}$

$f'(x) = 3 \cdot 2\, x^{\frac{1}{2}} + C$ (1)

$f'(4) = 3 \cdot 2 \cdot 2 + C$, $f'(4) = 7$ $\Rightarrow C = -5$ (2)

plug (2) into (1): $f'(x) = 6x^{\frac{1}{2}} - 5$

$\Rightarrow f(x) = 6 \cdot \frac{2}{3} \cdot x^{3/2} - 5x + C$ (3)

$f(4) = 4 \cdot 8 - 5 \cdot 4 + C = 20 \Rightarrow C = 8$ (4)

plug (4) into (3)

$f(x) = 4x^{3/2} - 5x + 8$

Example 4. Find the position of the particle given the following data:

$a(t) = t^2 - 4t + 6$, $s(1) = 20$, $s(0) = 0$.

$s''(t)$

$(s'(t))' = t^2 - 4t + 6$ take antiderivative

$\Rightarrow s'(t) = \frac{1}{3}t^3 - 4 \cdot \frac{1}{2}t^2 + 6t + C_1$ take antiderivative

$\Rightarrow s(t) = \frac{1}{3} \cdot \frac{1}{4}t^4 - 4 \cdot \frac{1}{2} \cdot \frac{1}{3}t^3 + 6 \cdot \frac{1}{2}t^2 + C_1 t + C_2$

Use the two initial conditions $s(1) = 20$, $s(0) = 0$ to determine C_1, C_2.

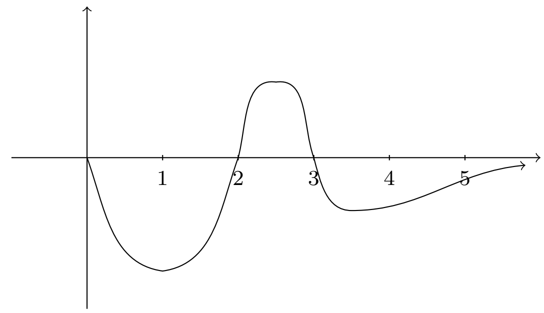
$20 = s(1) = \frac{1}{3} \cdot \frac{1}{4} \cdot 1 - 4 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 1 + 6 \cdot \frac{1}{2} \cdot 1 + C_1 \cdot 1 + C_2$

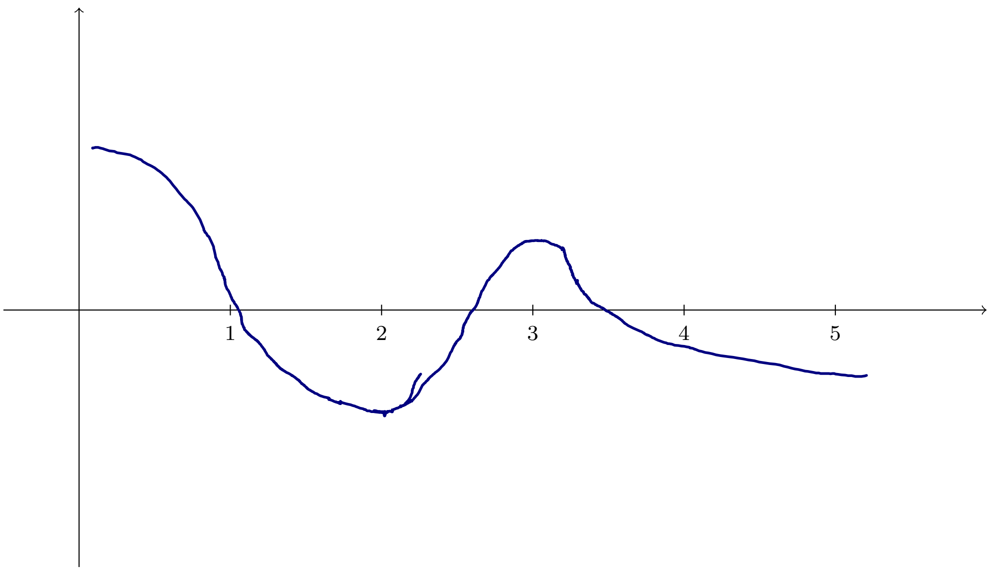
$0 = s(0) = 0 - 0 + 0 + 0 + C_2$

$\Rightarrow \begin{cases} C_2 = 0 \\ C_1 = 17\frac{7}{12} \end{cases}$

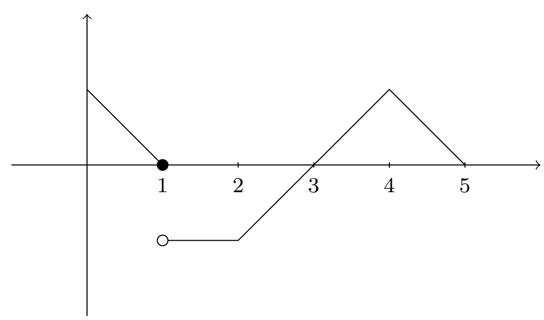
Graphing Problems

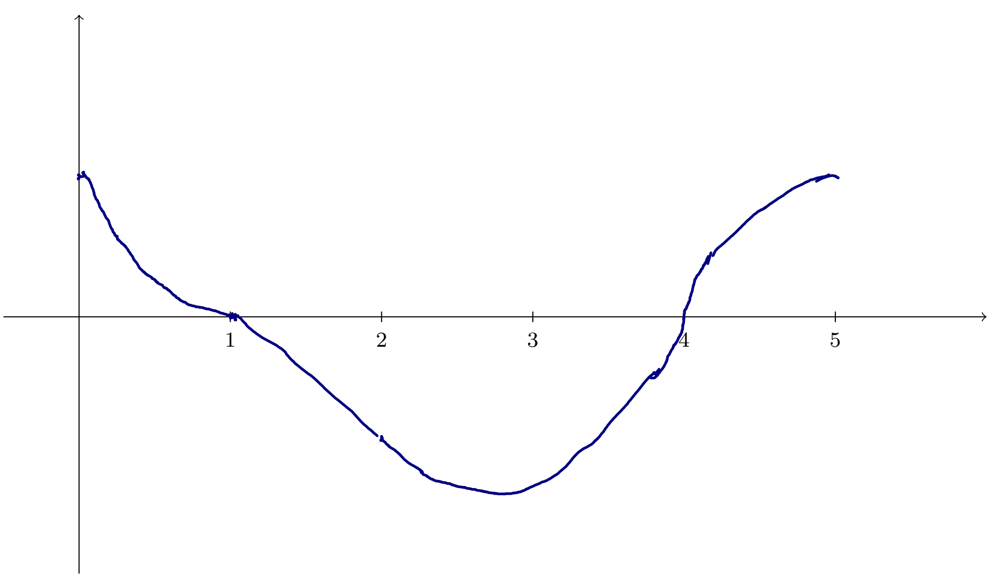
Example 5. Sketch an antiderivative of f below:





Example 6. Sketch an antiderivative of f below:





Tougher Problems

Example 7. Give an example of a derivative f' so that while solving the initial value problem with $f(0) = 0$ we have $C \neq 0$.

$$f'(x) = \sin x$$

$$f(x) = -\cos x + C$$

$$f(0) = -1 + C = 0$$

$$C = 1$$

since $C \neq 0$, this f' meet the requirement

Example 8. Given that the graph of f passes through the point $(1, 6)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$ find $f(2)$.

$f(1) = 6$ (1)

$f'(x) = 2x + 1$ (2)

antiderivative

$$f(x) = x^2 + x + C$$

$$6 = f(1) = 1 + 1 + C \Rightarrow C = 4$$

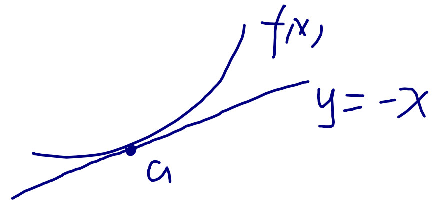
So $f(x) = x^2 + x + 4$

Example 9. Find a function f such that $f'(x) = x^3$ and $x + y = 0$ is a tangent line to the graph of f.

antiderivative

$$y = -x$$

$$f(x) = \frac{1}{4}x^4 + C$$



suppose $y = -x$ is tangent to $f(x)$ at a

from the figure, we can see this means

$$\left.\begin{array}{l} f(a) = -a \\ f'(a) = -1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{1}{4}a^4 + C = -a \\ a^3 = -1 \end{array}\right. \Rightarrow \begin{array}{l} a = -1 \\ C = \frac{5}{4} \end{array}$$