3.3 Problems Part 1

Derivatives and Graphs

Example 1. If you are given a formula for a function $f(x)$, how do you determine where f is increasing or decreasing?

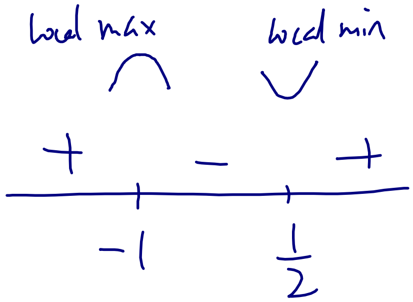
Example 2. State the first derivative test (without looking in your notes)

Example 3. For the following functions, find the intervals on which it is increasing and decreasing, and find where any local maximum and/or minimum values occur.

(a) $f(x) = 4x^3 + 3x^2 - 6x + 1$

$$f'(x) = 12x^2 + 6x - 6$$
$$= 6(2x^2 + x - 1)$$
$$= 6(2x-1)(x+1)$$

$x = \frac{1}{2}, -1$ are the critical points



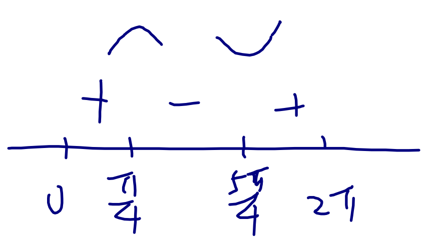
local max at $x = -1$

local min at $x = \frac{1}{2}$

(b) $f(x) = \sin x + \cos x$ on the domain $[0, 2\pi]$ (the one we did in class was $f(x) = \sin x - \cos x$)

$$f'(x) = \cos x - \sin x = 0$$
$$\Leftrightarrow \quad \tan x = 1$$
$$\Leftrightarrow \quad x = \frac{\pi}{4} + k\pi \text{ for any integer } k$$

among these, $\frac{\pi}{4}, \frac{5\pi}{4}$ are in $[0, 2\pi]$



local max at $x = \frac{\pi}{4}$

local min at $x = \frac{5\pi}{4}$

(c) $f(x) = x^{2/3}(6-x)^{1/3}$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}(6-x)^{1/3} - \frac{1}{3}x^{2/3}(6-x)^{-2/3}$$

$f'(x)$ is undefined at $x = 0, 6$

$f'(x) = 0$ when $\frac{2}{3}x^{-\frac{1}{3}}(6-x)^{1/3} - \frac{1}{3}x^{2/3}(6-x)^{-2/3} = 0$

$$\frac{2}{3}x^{-\frac{1}{3}}(6-x)^{1/3} = \frac{1}{3}x^{2/3}(6-x)^{-2/3}$$

multiplying both sides by $x^{1/3}(6-x)^{2/3}$ we get rid of the denominators

$$\frac{2}{3}(6-x) = \frac{1}{3}x \iff x = 4$$

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0 4 6

local max at 4

local min at 0

Example 4. Again consider the function from example 3(a): $f(x) = 4x^3 + 3x^2 - 6x + 1$

(a) Use the concavity test to find the intervals of concavity and the inflection points.

$$f'(x) = 12x^2 + 6x - 6$$

$$f''(x) = 24x + 6$$

$f''(x) > 0$ when $x > -\frac{1}{4}$: concave upward

$f''(x) < 0$ when $x < -\frac{1}{4}$: concave downward

inflection point : $x = \frac{1}{4}$

(b) Use your results from 3(a) and 4(a), along with the facts that

- f has a y-intercept at $y = 1$
- f has x-intercepts near $x = \underline{-1.7, 0.2, \text{ and } 0.8}$.

to sketch the graph of the function.

Recall from 4(a): $f''(x)$

inflection

$$\begin{array}{ccc} - & & + \\ \hline & -\frac{1}{4} & \end{array}$$

from 3(a): $f'(x)$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & \frac{1}{2} & \end{array}$$

local max   local min