Homework 9

Problem 1 (3pts)**.** *Let*

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

1. *Compute* $\|A\|_F$ *directly from the entries of* A*.*
2. *Find the singular values of* A *and verify that*

$$\|A\|_F^2 = \sum_j \sigma_j^2, \qquad \|A\|_{2\to 2} = \sigma_1.$$

Problem 2 (3pts)**.** *Let* $A \in \mathbb{C}^{m\times n}$ *have rank* $r > 0$ *and singular values*

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0.$$

Prove that

$$\|A\|_{2\to 2} \leq \|A\|_F \leq \sqrt{r}\, \|A\|_{2\to 2}\,.$$

When does

$$\|A\|_{2\to 2} = \|A\|_F$$

hold?

Problem 3 (4pts)**.** *Suppose*

$$A = 8u_1v_1^* + 4u_2v_2^* + 3u_3v_3^* + u_4v_4^*$$

is the rank-one expansion of an SVD.

1. *Write down the rank-2 truncated SVD* A_2*.*
2. *Compute*

$$\|A - A_2\|_{2\to 2} \qquad \textit{and} \qquad \|A - A_2\|_F\,.$$

3. *Among all matrices* B *with*

$$\operatorname{rank}(B) \leq 2,$$

what are the smallest possible values of

$$\|A - B\|_{2\to 2} \qquad \textit{and} \qquad \|A - B\|_F?$$

4. *Find the smallest* s *such that*

$$\|A - A_s\|_{2\to 2} \leq 3.$$

Then find the smallest s *such that*

$$\|A - A_s\|_F \leq 3.$$