Homework 8

Problem 1 (2pts)**.** *Let*

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 5 & 3 \\ 2 & 3 & 3 \end{pmatrix}.$$

Find a Cholesky decomposition (without using QR)

$$A = LL^T,$$

where L is lower triangular.

Problem 2 (4pts)**.** *Let*

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}.$$

1. *Compute a compact singular value decomposition*

$$A = U_r \Sigma_r V_r^*.$$

2. *Use the SVD to give orthonormal bases for*

$$\mathcal{C}(A), \qquad \mathcal{N}(A), \qquad \mathcal{C}(A^*), \qquad \mathcal{N}(A^*).$$

Problem 3 (4pts)**.** *In class, we constructed the singular value decomposition of*

$$A \in \mathbb{C}^{m \times n}$$

using the eigenvalues and eigenvectors of A^*A.

This problem shows that the SVD can be constructed in a completely symmetric way using the eigenvalues and eigenvectors of AA^* instead.

Suppose

$$AA^*u = \lambda u, \qquad \lambda > 0, \qquad \|u\|_2 = 1.$$

Define

$$\sigma = \sqrt{\lambda}, \qquad v = \frac{A^*u}{\sigma}.$$

Show that this construction indeed gives a singular vector pair; that is, prove that

$$\boxed{Av = \sigma u,}$$

and that

$$\|v\|_2 = 1.$$