Homework 7

Problem 1 (5pts)**.** *Let*

$$A = \begin{pmatrix} 2 & i & 0 \\ -i & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in \mathbb{C}^{3\times 3}.$$

1. *Verify that A is Hermitian.*
2. *Find all eigenvalues of A and a basis for each eigenspace.*
3. *Normalize your eigenvectors and verify that eigenvectors corresponding to distinct eigenvalues are orthogonal with respect to the complex inner product*

$$\langle x, y\rangle = x^* y.$$

4. *Use your answer to determine*

$$\mathcal{N}(A) \qquad \textit{and} \qquad \operatorname{rank}(A).$$

Problem 2 (5pts)**.** *Let $A \in \mathbb{C}^{n\times n}$ be Hermitian and suppose*

$$A^2 = A.$$

1. *Show that every eigenvalue of A is either 0 or 1.*
2. *Show that*

$$\mathcal{N}(A) = \{x : Ax = 0\}, \qquad \mathcal{C}(A) = \{x : Ax = x\}.$$

3. *Prove that*

$$\mathcal{N}(A) = \mathcal{C}(A)^{\perp}.$$

4. *Explain why A is the orthogonal projection onto $\mathcal{C}(A)$.*