

Homework 4

Problem 1 (3pts). Let $Q \in \mathbb{R}^{m \times n}$ with $m \geq n$ be an orthogonal matrix; that is, the columns of Q are mutually orthogonal and each has unit norm. Let $x, y \in \mathbb{R}^m$. Show that

$$(I_m - QQ^T)x = x - QQ^T x$$

is orthogonal to $QQ^T y$.

Problem 2 (2pts). Let $U \in \mathbb{R}^{n \times n}$ be an orthogonal matrix. Compute its Frobenius norm $\|U\|_F$.

Problem 3 (2pts). Let $L, K \subset \mathbb{R}^n$ be two linear subspaces of $\mathbb{R}^n$ with

$$\dim(L) + \dim(K) > n.$$

Prove that $L \cap K$ is a linear subspace of $\mathbb{R}^n$ with

$$\dim(L \cap K) \geq 1.$$

Problem 4 (3pts). Find the QR decomposition for $A \in \mathbb{R}^{4 \times 4}$:

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & -1 \end{pmatrix}$$