Homework 3

Problem 1 (3pts). Let $S, T \subset \mathbb{R}^n$. Verify that

$$\operatorname{span}(T \cap S) \subset \operatorname{span}(T) \cap \operatorname{span}(S).$$

Problem 2 (3pts). Show that the rank of a matrix $A \in \mathbb{R}^{m \times n}$ always satisfies

$$\operatorname{rank}(A) \leq \min\{m, n\}.$$

Problem 3 (4pts). Run the Gram–Schmidt process by hand on the set

$$S = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \right\} \subset \mathbb{R}^3.$$

Verify that the basis $B \subset \mathbb{R}^3$ produced by the Gram–Schmidt process for $\operatorname{span}(S)$ is indeed an orthonormal basis.