Midterm Review

MTH 415 — Fall 2026

What You Should Know

- **Feed-Forward Neural Networks**
  - How to read a network diagram and write the corresponding function.
  - How weights, biases, and activation functions determine each layer.
  - How to evaluate the output of a small network for a given input.
- **ReLU Networks and Piecewise Linear Functions**
  - How a function of the form

$$f(x) = \sum_j c_j \,\mathrm{ReLU}(w_j x + b_j)$$

    behaves.
  - How to identify the locations of kinks.
  - How different ReLU terms can create or cancel kinks.
  - How to represent simple piecewise linear functions using ReLU networks.
- **Constructing Simple Neural Networks**
  - How to choose an activation function appropriate for a target function.
  - How to construct a small network representing a given scalar- or vector-valued function.
  - How to determine weights and biases from the desired functional form.
- **Subspaces**
  - How to verify that a set is a subspace.
  - Sums and intersections of subspaces.
  - Dimension relations involving $L + K$ and $L \cap K$.
- **Span, Linear Independence, Basis, and Dimension**
  - How these concepts are related.
  - How to determine whether a given set is a basis.
  - How to find the dimension of a subspace.
- **Column Space, Null Space, and Rank**
  - How to find bases for $\mathcal{C}(A)$ and $\mathcal{N}(A)$.
  - Rank and nullity.
  - The relationship between rank and linear independence of columns.
- **Inner Products, Norms, and Orthogonality**

- How to compute inner products and norms.
- How to check whether vectors are orthogonal or orthonormal.
- Basic properties of orthonormal sets.

- **Gram–Schmidt**
  - How to convert a linearly independent set into an orthonormal basis.
  - How Gram–Schmidt relates to QR decomposition.

- **QR Decomposition**
  - How to compute a reduced QR decomposition.
  - The roles of Q and R.
  - What changes when the matrix is rank deficient.

- **Orthogonal Projection**
  - Projection onto a subspace with an orthonormal basis.
  - Difference between Q^TQ and QQ^T.
  - Be careful whether basis vectors are stored as columns or rows.
  - Orthogonality of the residual to the subspace.

- **Least Squares**
  - Geometric interpretation as projection onto $\mathcal{C}(A)$.
  - Residual orthogonality condition.
  - Normal equations.
  - Solving least squares using QR.
  - When the least-squares solution is unique.

- **Four Fundamental Subspaces**
  - $\mathcal{C}(A)$, $\mathcal{N}(A)$, $\mathcal{C}(A^T)$, and $\mathcal{N}(A^T)$.
  - Which ambient space each subspace belongs to.
  - Orthogonality relationships between them.
  - Dimensions of the four subspaces.
  - $\text{rank}(A) = \text{rank}(A^T)$.