3.7 Problems

Example 1. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.

(a) Find the demand function, assuming that it is linear.

$y = mx + b$ $\quad (10, 27000)$, $(8, 33000)$ use these points to find m and b.

y ↑ demand, x ↑ price

$$m = \frac{33000 - 27000}{8 - 10} = -3000$$

$$27000 = -3000 \cdot 10 + b \Rightarrow b = 57000$$

$$y = -3000x + 57000 \qquad (1)$$

(b) How should ticket prices be set to maximize revenue?

R: revenue

$$R = x \cdot y \qquad (x \geq 0,\ 0 \leq y \leq 55000)$$

$$= x \cdot (-3000x + 57000) \qquad x \in [\tfrac{2}{3}, 19]$$

$$= -3000x^2 + 57000x$$

$y \geq 0$ and (1) imply

$$-3000x + 57000 \geq 0$$

$$\Leftrightarrow \underline{x \leq 19}$$

$y \leq 55000$ and (1) imply

$$-3000x + 57000 \leq 55000$$

$$\Leftrightarrow \underline{x \geq 2/3}$$

$$R' = -6000x + 57000$$

critical point is $\frac{57000}{6000} = 9.5$

$R(9.5) = \underline{27075}0$ ← largest

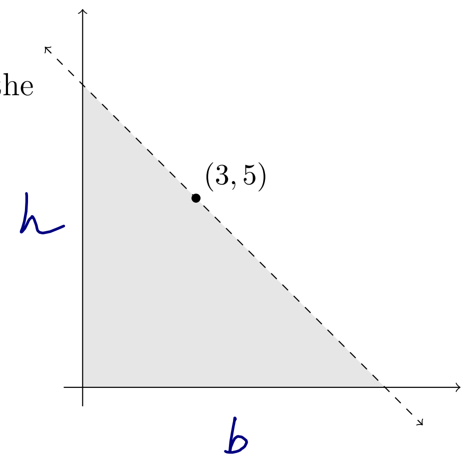
$R(\frac{2}{3}) \approx 36000$

$R(19) = 0$

price should be set to 9.5

Example 2.

Find an equation of the line through the point $(3,5)$ that cuts off the least area from the first quadrant.



A : shaded area

$$A = \frac{1}{2} b h \qquad (1)$$

b and h are the x and y intercepts of the line. since the line passes (3,5), it has the expression

$$y - 5 = m(x-3)$$

$$\Leftrightarrow \quad y = mx - 3m + 5 \qquad (2)$$

its y-intercept is $-3m+5$, x-intercept is $\frac{3m-5}{m}$

so $b = \frac{3m-5}{m}$, $h = -3m+5$ plugging these into (1)

$$A = \frac{1}{2}(-3m+5) \cdot \frac{3m-5}{m}$$

$$= -\frac{1}{2} \frac{(3m-5)^2}{m}$$

$$= -\frac{1}{2}\left(9m - 30 + \frac{25}{m}\right)$$

$$A' = -\frac{1}{2}\left(9 - \frac{25}{m^2}\right)$$

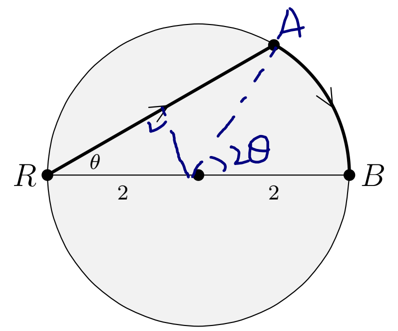
critical point $m = \cancel{\frac{5}{3}}, -\frac{5}{3}$

insert $m = -5/3$ into (2), we get the line equation

$$y = -\frac{5}{3}x + 10$$

Example 3.

Ryan is at point R on the shore of a circular lake with radius 2 mi and all of a sudden really has to use the bathroom which is at point B diametrically opposite R (see the picture to the right). He can run at the rate of 8 mi/h and row a boat at 4 mi/h. How should he proceed?



time to boat from R to A : $\dfrac{\text{length of } RA}{4}$

↑ speed of boating

time to run from A to B : $\dfrac{\text{arc length of } \widehat{AB}}{8}$

↑ speed of running

T: total time

$$T = \frac{\text{length of } RA}{4} + \frac{\text{arc length of } \widehat{AB}}{8} \qquad (1)$$

In terms of θ : length of $RA = 2 \cdot \cos\theta \cdot 2 = 4\cos\theta$ (2)

arc length of $\widehat{AB} = 2 \cdot 2\theta = 4\theta$ (3)

plug (2) (3) into (1) :

$$T = \frac{4\cos\theta}{4} + \frac{4\theta}{8} = \cos\theta + \frac{\theta}{2}$$ (by definition of θ, $0 \le \theta \le \pi/2$)

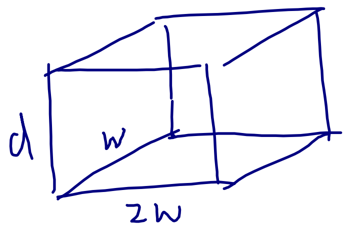
$T' = -\sin\theta + \frac{1}{2}$

so critical point is : $\sin\theta = \frac{1}{2}$

$\iff \theta = \frac{\pi}{6}$

so he should boat towards an angle of $\frac{\pi}{6}$, and run to B once arriving at A.

Example 4. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of the materials for the cheapest such container.



C: cost

base area

area of the 4 sides

$$C = 10\cdot(w\cdot 2w) + 6\cdot(6dw) \quad (1)$$

cost per square meter for the base

cost per square meter for the sides

volume is 10 means

$$10 = d\cdot w\cdot 2w$$

$$\Rightarrow \quad d = \frac{10}{2w^2} = \frac{5}{w^2} \quad (2)$$

plug (2) into (1)

$$C = 20w^2 + \frac{36\cdot 5}{w}$$

$$C' = 40w - \frac{180}{w^2}$$

critical point is $\left(\frac{9}{2}\right)^{1/3}$

$C\left(\left(\frac{9}{2}\right)^{1/3}\right) \approx 163.54 \leftarrow$ cheapest