3.3 Problems Part 2

Give Examples of...

Question 1. If possible, give an example of a function f that satisfies:

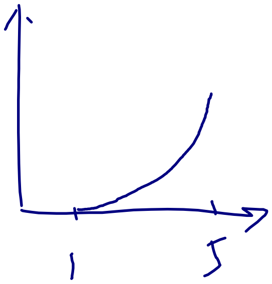
- f is decreasing on $[1, 5]$
- f is concave up on $[1, 5]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.



$y = (x-1)^2$

Question 2. If possible, give an example of a function f that satisfies:

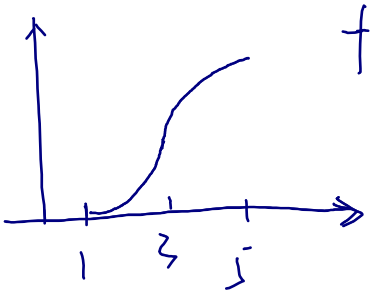
- f is increasing on $[1, 5]$
- f is concave up on $[1, 3)$ and is concave down on $(3, 5]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.



$$f(x) = \begin{cases} (x-1)^2 & \text{if } 1 \le x \le 3 \\ -(x-5)^2 + 8 & \text{if } 3 \le x \le 5 \end{cases}$$

Question 3. If possible, give an example of a function f that satisfies:

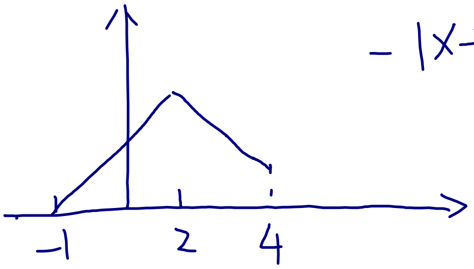
- f is increasing on $[-1, 2)$ and decreasing on $(2, 4]$.
- f is neither concave up or concave down on $[-1, 4]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.



$-|x-2|+3$

Question 4. If possible, give an example of a function f that satisfies:

- f is neither increasing nor decreasing on $[1, 4]$.
- f is concave up on $[1, 4]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.

Not possible

Because if $f'(x) = 0$ on $[1,4]$, then

$f(x) = \text{constant}$, this f is not concave up

Question 5. Consider the function $f(x) = \frac{1}{x} + \frac{x}{16}$. Find where:

(a) f is increasing/decreasing

domain $(-\infty, 0) \cup (0, \infty)$

(b) f is concave up/down

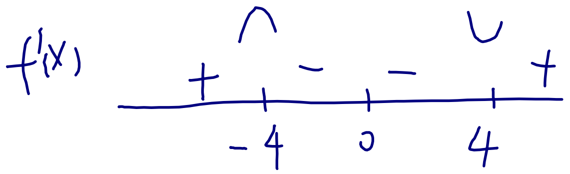
(c) Identify any local mins, local maxes, and inflection points.

(a) $f'(x) = -x^{-2} + \frac{1}{16}$

$f'(x)$ is undefined at 0

$f'(x) = 0$ when $-x^{-2} + \frac{1}{16} = 0$

$\Rightarrow x = \pm 4$

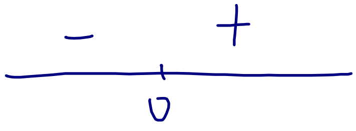


$f(x)$ is increasing on $(-\infty, -4) \cup (4, \infty)$

decreasing on $(-4, 0) \cup (0, 4)$

(b) $f''(x) = 2x^{-3}$

undefined at 0



concave up on $(0, \infty)$

concave down on $(-\infty, 0)$

(c) local max at $x = -4$

local min at $x = 4$

no inflection point (as 0 is not in the domain)

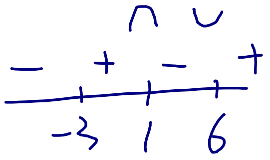
Question 6. Given

$$f'(x) = \frac{(x-6)(x-1)}{(x+3)}, \qquad f''(x) = \frac{(x+9)(x-3)}{(x+3)^3}, \qquad f(-3) \text{ not defined.}$$

determine the intervals on which $f(x)$ increases/decreases, the intervals on which the function is concave up/down and the x values in which the function has maximum, minimum and inflection.

$f'(x)=0$ when $x=1,6$

$f'(x)$ is undefined at $x=-3$



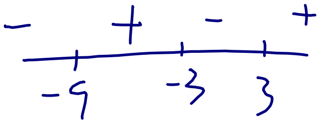
Local min at 6

Local max at 1

−3 is not in the domain

$f''(x)=0$ when $x=3,-9$

$f''(x)$ is undefined at −3



inflection points at

$x=-9, 3$

−3 is not in the domain thus it's not an inflection point

Question 7. Determine A and B so that the curve $y = Ax^{1/9} + Bx^{-1/9}$ has an inflection point at $(1, 2)$.

plug (1, 2) into the equation

$2 = A1^{1/9} + B1^{-1/9}$

$\Leftrightarrow 2 = A+B$ ①

The curve has an inflection point at (1, 2), so

$y''(1) = 0$ or unde~~fined~~

$y'(x) = \frac{1}{9}Ax^{-8/9} - \frac{1}{9}Bx^{-10/9}$

$y''(x) = \frac{1}{9}\cdot(-\frac{8}{9})Ax^{-17/9} + \frac{1}{9}\cdot\frac{10}{9}\cdot Bx^{-19/9}$

this expression tells us $y''(x)$ is only undefined at $x=0$, $y''(1)$ is defined

so $y''(1)=0 \Rightarrow -\frac{8}{81}A\cdot 1^{-17/9} + \frac{10}{81}B\cdot 1^{-19/9} = 0 \Leftrightarrow 4A = 5B$ ②

Solve ①.②. we get $B=\frac{8}{9}$ $A=\frac{10}{9}$