Exercise 9.2

First we recall that for Makeham's, we have

$$_t p_x = \exp\left\{-\left[At + \frac{Bc^x}{\log(c)}(c^t - 1)\right]\right\}.$$

(a)
$$
\begin{aligned}
_{10}p_{30:40} &= {_{10}p_{30}} \times {_{10}p_{40}} \\
&= \exp\left\{-\left[10A + \frac{Bc^{30}}{\log(c)}(c^{10} - 1)\right]\right\} \times \exp\left\{-\left[10A + \frac{Bc^{40}}{\log(c)}(c^{10} - 1)\right]\right\} \\
&= 0.9612375 \times 0.9227294 \\
&= 0.8869621,
\end{aligned}
$$

after plugging the given values for A, B and c.

(b)
$$
\begin{aligned}
_{10}q^1_{30:40} &= \int_0^{10} {_t p_{30}}\, \mu_{30+t} \times {_t p_{40}}\, dt \\
&= 0.03725698.
\end{aligned}
$$

(c)
$$
\begin{aligned}
_{10}q^2_{30:40} &= \int_0^{10} {_t p_{40}}\, \mu_{40+t} \times {_t p_{30}}\, dt \\
&= 0.07578088.
\end{aligned}
$$

This answer does not match the one printed in the textbook. But this can easily be verified. Note that parts (b) and (c) should clearly sum to $_{10}q_{30:40}$ which is equal to

$$_{10}q_{30:40} = 1 - {_{10}p_{30:40}} = 1 - 0.8869621 = 0.1130379,$$

which should be the sum $0.03725698 + 0.07578088 = 0.1130379$. The textbook answer is clearly incorrect.

(d)
$$
\begin{aligned}
_{10}p_{\overline{30:40}} &= {_{10}p_{30}} + {_{10}p_{40}} - {_{10}p_{30}} \times {_{10}p_{40}} \\
&= 0.9612375 + 0.9227294 - 0.9612375 \times 0.9227294 \\
&= 0.9970048.
\end{aligned}
$$

For those interested in the simple R code to evaluate the integrals above, we have

```
x <- 30
y <- 40
A <- 0.0001
B <- 0.0003
c <- 1.075
mu30 <- function(t){
out <- A + B*c^(x+t)
out}
mu40 <- function(t){
out <- A + B*c^(y+t)
out}
```

```
tp30 <- function(t){
temp <- A*t + B*c^x*(c^t - 1)/log(c)
out <- exp(-temp)
out}
tp40 <- function(t){
temp <- A*t + B*c^y*(c^t - 1)/log(c)
out <- exp(-temp)
out}

proba <- tp30(10)*tp40(10)

h <- 1/1000
t <- seq(0,10,h)
intb <- tp30(t)*mu30(t)*tp40(t)
probb <- 0
n <- 1
while (n<length(t)) {
n <- n+2
probb <- probb + (h/3)*(intb[n-2]+4*intb[n-1]+intb[n])
}

h <- 1/1000
t <- seq(0,10,h)
intc <- tp40(t)*mu40(t)*tp30(t)
probc <- 0
n <- 1
while (n<length(t)) {
n <- n+2
probc <- probc + (h/3)*(intc[n-2]+4*intc[n-1]+intc[n])
}

probd <- tp30(10) + tp40(10) - proba

> tp30(10)
[1] 0.9612375
> tp40(10)
[1] 0.9227294
> proba
[1] 0.8869621
> probb
[1] 0.03725698
> probc
[1] 0.07578088
> 1-proba
[1] 0.1130379
> probb + probc
```

```
[1] 0.1130379
> probd <- tp30(10) + tp40(10) - proba
> probd
[1] 0.9970048
```