

Exercise 9.16

(a) The APV of the death benefits is given by

$$\begin{aligned} \text{APV}(\text{DB}) &= 10000\bar{A}_{65:60} + 1000\bar{A}_{\overline{65:60}} \\ &= 10000\bar{A}_{65:60} + 1000(\bar{A}_{65} + \bar{A}_{60} - \bar{A}_{65:60}) \\ &= 10000(0.512589) + 1000(0.473229 + 0.353789 - 0.512589) \\ &= 5440.319 \end{aligned}$$

(b) The APV of the reversionary benefits is given by

$$\begin{aligned} \text{APV}(\text{RB}) &= 5000\bar{a}_{60|65} + 5000\bar{a}_{65|60} \\ &= 5000(\bar{a}_{65} + \bar{a}_{60} - 2 \cdot \bar{a}_{65:60}) \\ &= 5000 \left[\frac{1}{\delta}(1 - \bar{A}_{65}) + \frac{1}{\delta}(1 - \bar{A}_{60}) - 2 \cdot \frac{1}{\delta}(1 - \bar{A}_{65:60}) \right] \\ &= 5000(13.43094 + 16.47627 - 2 \cdot 12.42739) \\ &= 25262.16 \end{aligned}$$

(c) Let P denote the annual premium rate payable continuously.

$$P = \frac{\text{APV}(\text{DB}) + \text{APV}(\text{RB})}{\bar{a}_{65:60}} = \frac{5440.319 + 25262.16}{12.42739} = 2470.55$$

(d) For the 10-th year policy value,

(i) if both are still alive:

$$\begin{aligned} {}_{10}V &= [\text{APV}(\text{DB}_{10}) + \text{APV}(\text{RB}_{10})] - P\bar{a}_{75:70} \\ &= 1000 \times [10\bar{A}_{75:70} + \bar{A}_{\overline{75:70}} + 5(\bar{a}_{70|75} + \bar{a}_{75|70})] - P\bar{a}_{75:70} \end{aligned}$$

(ii) if husband is dead, but wife still alive (the superscript f to emphasize survival of wife):

$${}_{10}V = 1000\bar{A}_{70}^f + 5000\bar{a}_{70}^f$$

(iii) Thiele's differential equations satisfy:

$$\frac{d}{dt} {}_tV^{(0)} = \delta {}_tV^{(0)} + P - \mu_{65+t:60+t}^{01}(10000 + {}_tV^{(1)}) - \mu_{65+t:60+t}^{02}(10000 + {}_tV^{(2)}),$$

if both are still alive, and

$$\frac{d}{dt} {}_tV^{(2)} = \delta {}_tV^{(2)} - \mu_{60+t}^{23}(1000 - {}_tV^{(2)}),$$

if the husband is dead while the wife is still alive. The states are as defined as in Figure 9.2 (DHW, 2nd ed.).