Exercise 8.9

First, we note that because $\mu_x^{02} = \mu_x^{12}$ for all x, we have

$$\begin{aligned} {}_tp_x^{00}\ {}_{n-t}p_{x+t}^{11} &= e^{-\int_0^t \left(\mu_{x+s}^{01}+\mu_{x+s}^{02}\right)ds} e^{-\int_0^{n-t} \mu_{x+t+z}^{12} dz} \\ &= e^{-\int_0^t \mu_{x+s}^{01} ds} e^{-\left(\int_0^t \mu_{x+s}^{02} ds + \int_t^n \mu_{x+s}^{02} ds\right)} \\ &= e^{-\int_0^t \mu_{x+s}^{01} ds} e^{-\int_0^n \mu_{x+s}^{02} ds} \end{aligned}$$

giving us the desired result. Now, starting with

$$\begin{aligned} {}_np_x^{01} &= \int_0^n {}_tp_x^{00}\, \mu_{x+t}^{01}\ {}_{n-t}p_{x+t}^{11} dt \\ &= \int_0^n e^{-\int_0^t \mu_{x+s}^{01} ds} e^{-\int_0^n \mu_{x+s}^{02} ds}\, \mu_{x+t}^{01} dt \\ &= e^{-\int_0^n \mu_{x+s}^{02} ds} \int_0^n e^{-\int_0^t \mu_{x+s}^{01} ds}\, \mu_{x+t}^{01} dt \\ &= e^{-\int_0^n \mu_{x+s}^{02} ds} \left(1 - e^{-\int_0^n \mu_{x+s}^{01} ds}\right). \end{aligned}$$