Exercise 7.9

(a) Let $T_{[x]+t}$ denote the future lifetime of a life with select age x, t years later. Therefore, the insurer's loss at duration t for this life can be expressed as

$$L_t^n = Sv^{T_{[x]+t}} - P\bar{a}_{\overline{T_{[x]+t}}|} = \left(S + \frac{P}{\delta}\right) v^{T_{[x]+t}} - \frac{P}{\delta}.$$

The expectation of this loss at duration t can be expressed as

$$\mathrm{E}[L_t^n] = S\bar{A}_{[x]+t} - P\bar{a}_{[x]+t} = \left(S + \frac{P}{\delta}\right) \bar{A}_{[x]+t} - \frac{P}{\delta}$$

and the variance is

$$\begin{aligned} \mathrm{Var}[L_t^n] &= \left(S + \frac{P}{\delta}\right)^2 \mathrm{Var}\left[v^{T_{[x]+t}}\right] \\ &= \left(S + \frac{P}{\delta}\right)^2 \left[{}^2\bar{A}_{[x]+t} - \left(\bar{A}_{[x]+t}\right)^2\right]. \end{aligned}$$

(b) For $x = 55$ and $P = 1200$, by the equivalence principle, we have

$$S = P \times \frac{\bar{a}_{[55]}}{\bar{A}_{[55]}} = 1200 \times \frac{15.56159}{0.2407473} = 77566.44.$$

Here, as is the case with the rest of the problem, $\bar{a}_{[x]}$ is estimated using repeated Simpson's rule with $h = 1/100$.

(c) One can verify the following calculations of the variances, and corresponding standard deviations, of the losses at durations 0, 5, and 10:

$$\begin{aligned} \mathrm{Var}[L_0^n] &= \left(S + \frac{P}{\delta}\right)^2 \left[{}^2\bar{A}_{[55]} - \left(\bar{A}_{[55]}\right)^2\right] \\ &= \left(77566.44 + \frac{1200}{\log(1.05)}\right)^2 \left[0.07821618 - (0.2407473)^2\right] = 211421004 \end{aligned}$$

so that the standard deviation is

$$\mathrm{SD}[L_0^n] = \sqrt{211421004} = 14540.32.$$

$$\begin{aligned} \mathrm{Var}[L_5^n] &= \left(S + \frac{P}{\delta}\right)^2 \left[{}^2\bar{A}_{[55]+5} - \left(\bar{A}_{[55]+5}\right)^2\right] \\ &= \left(77566.44 + \frac{1200}{\log(1.05)}\right)^2 \left[0.1137389 - (0.2974343)^2\right] = 263760954 \end{aligned}$$

so that the standard deviation is

$$\mathrm{SD}[L_5^n] = \sqrt{263760954} = 16240.72.$$

$$\begin{aligned}\mathrm{Var}[L_{10}^n] &= \left(S + \frac{P}{\delta}\right)^2 \left[{}^2\bar{A}_{[55]+10} - \left(\bar{A}_{[55]+10}\right)^2\right] \\ &= \left(77566.44 + \frac{1200}{\log(1.05)}\right)^2 \left[0.1618931 - (0.3635198)^2\right] = 310463852\end{aligned}$$

so that the standard deviation is

$$\mathrm{SD}[L_{10}^n] = \sqrt{310463852} = 17619.98.$$

Thus, we observe an increasing variation in the loss with duration, partially due to the increasing variation of the timing of when the death payment is to be made.