Exercise 7.8

(a) Based on the equivalence principle, the net premium per year payable continuously can be expressed as

$$P = 200000 \times \frac{\bar{A}_{[40]:\overline{20}|}}{\bar{a}_{[40]:\overline{20}|}},$$

where

$$\bar{a}_{[40]:\overline{20}|} = \int_0^{20} v^t \, {}_tp_{[40]} \, dt = 12.67553$$

and

$$\bar{A}_{[40]:\overline{20}|} = 1 - \log(1.05)(12.67553) = 0.3815587.$$

Thus, it follows that

$$P = 200000 \times \frac{0.3815587}{12.67553} = 6020.398.$$

Here, we note that $\bar{a}_{[40]:\overline{20}|}$ has been approximated based on repeated Simpson's rule with $h = 1/100$.

(b) For a policy still in force at duration 4, the policy value at that time can be expressed as

$$\begin{aligned} {}_4V &= \mathrm{APV}(\mathrm{FB}_4) - \mathrm{APV}(\mathrm{FP}_4) \\ &= 200000 \times \bar{A}_{44:\overline{16}|} - P \times \bar{a}_{44:\overline{16}|} \\ &= 200000(0.4623625) - 6020.398(11.01938) = 26131.42. \end{aligned}$$

Here, $\bar{a}_{[40]:\overline{20}|}$ is similarly approximated based on repeated Simpson's rule with $h = 1/100$.

(c) Revising the value of A to 0.0004 and holding all other assumptions, the (revised) policy value at duration 4 is

$$\begin{aligned} {}_4V &= \mathrm{APV}(\mathrm{FB}_4) - \mathrm{APV}(\mathrm{FP}_4) \\ &= 200000 \times \bar{A}_{44:\overline{16}|} - P \times \bar{a}_{44:\overline{16}|} \\ &= 200000(0.4630335) - 6020.398(11.00563) = 26348.41. \end{aligned}$$

(d) Increasing the value of A in the Makeham's law has the effect of worsening mortality which, not surprisingly in this case, potentially increased the policy value. The effect of a mortality revision is not significant in this case.

(e) Revising the value of i from 5% to 4% and holding all other assumptions, the (revised) policy value at duration 4 is

$$\begin{aligned} {}_4V &= \text{APV}(\text{FB}_4) - \text{APV}(\text{FP}_4) \\ &= 200000 \times \bar{A}_{44:\overline{16}|} - P \times \bar{a}_{44:\overline{16}|} \\ &= 200000(0.5376978) - 6020.398(11.7872) = 36575.95. \end{aligned}$$

(f) Clearly in this case, when interest rates earned on assets are lower than assumed, assets backing the reserves will grow at a much lower pace requiring therefore to hold much larger reserves than originally assumed. The effect of a change in interest rate is more dramatic than the effect of a mortality change, as previously observed.

(g) The (original) policy value, calculated based on the original set of assumptions, can be expressed as

$${}_kV = 200000 \times \bar{A}_{[40]+k:\overline{20-k}|} - P \times \bar{a}_{[40]+k:\overline{20-k}|}.$$

Based on the proposed contract alteration of a proportionate paid-up sum insured, the (revised) policy value will be calculated as

$${}_kV(\text{RPU}) = 200000 \times (k/20) \times \bar{A}_{[40]+k:\overline{20-k}|},$$

where RPU is to indicate "reduced paid-up" policy. Note that for such a policy, no future premiums are to be paid at duration k.

We compare these two policy values on a tabular basis as well as graphically in the following.

k	Original ${}_kV$	Proportionate Reduced Paid-Up ${}_kV(\text{RPU})$	k	Original ${}_kV$	Proportionate Reduced Paid-Up ${}_kV(\text{RPU})$
0	0.000	0.000			
1	6068.801	4003.556	11	85896.319	71188.467
2	12447.536	8400.982	12	96212.173	81487.995
3	19124.787	13220.896	13	107044.355	92633.056
4	26131.416	18494.499	14	118421.016	104683.733
5	33483.770	24254.873	15	130372.436	117704.582
6	41199.091	30537.290	16	142931.311	131765.095
7	49295.588	37379.361	17	156133.076	146940.252
8	57792.520	44821.211	18	170016.304	163311.161
9	66710.287	52905.664	19	184623.167	180965.816
10	76070.542	61678.455	20	200000.000	200000.000

Note that the policy value at duration 10 is 76070.542, as opposed to \$70070.54 as printed in the answers in the DHW textbook.

The graphical comparison of the policy values between the original policy and that of the reduced paid-up indicates that, as reasonably should be the case, the reduced paid-up always yield a lower policy value. Generally, for life insurance contracts, early surrender of policies is highly discouraged which does not appear to be in this situation.