Exercise 7.5

(a) Let P denote the annual premium rate payable continuously. The present value of future benefits at issue can be expressed as

$$\text{PVFB}_0 = \begin{cases} T_{30} P v^{T_{30}}, & \text{for } T_{30} \leq 10 \\ 10 P v^{T_{30}}, & \text{for } 10 < T_{30} \leq 30 \\ v^{30} \ddot{a}^{(12)}_{\overline{K^{(12)}_{30} + (1/12) - 30}|}, & \text{for } T_{30} > 30 \end{cases}$$

The present value of future premiums at issue can be expressed as

$$\text{PVFP}_0 = \begin{cases} P \bar{a}_{\overline{T_{30}}|}, & \text{for } T_{30} \leq 10 \\ P \bar{a}_{\overline{10}|}, & \text{for } T_{30} > 10 \end{cases}$$

The future loss random variable L_0 is the difference between the two:

$$L_0 = \text{PVFB}_0 - \text{PVFP}_0 = \begin{cases} T_{30} P v^{T_{30}} - P \bar{a}_{\overline{T_{30}}|}, & \text{for } T_{30} \leq 10 \\ 10 P v^{T_{30}} - P \bar{a}_{\overline{10}|}, & \text{for } 10 < T_{30} \leq 30 \\ v^{30} \ddot{a}^{(12)}_{\overline{K^{(12)}_{30} + (1/12) - 30}|} - P \bar{a}_{\overline{10}|}, & \text{for } T_{30} > 30 \end{cases}$$

(b) The actuarial present value of future premiums at issue is

$$\text{APV(FP)} = P \bar{a}_{30:\overline{10}|}.$$

The actuarial present value of future benefits at issue is

$$\text{APV(FB)} = P \big(\bar{I}\bar{A}\big)^{\;1}_{30:\overline{10}|} + 10P\, {}_{10}E_{30}\, \bar{A}^{\;1}_{40:\overline{20}|} + {}_{30}E_{30}\, \ddot{a}^{(12)}_{60}.$$

Equating the two APV's by the equivalence principle, we get

$$P = \frac{{}_{30}E_{30}\, \ddot{a}^{(12)}_{60}}{\bar{a}_{30:\overline{10}|} - \big(\bar{I}\bar{A}\big)^{\;1}_{30:\overline{10}|} - 10\, {}_{10}E_{30}\, \bar{A}^{\;1}_{40:\overline{20}|}}.$$

(c) For a policy still in force at duration 5, the present value of future loss random variable can be expressed as

$$L_5 = \text{PVFB}_5 - \text{PVFP}_5 = \begin{cases} (5 + T_{35}) P v^{T_{35}} - P \bar{a}_{\overline{T_{35}}|}, & \text{for } T_{35} \leq 5 \\ 10 P v^{T_{35}} - P \bar{a}_{\overline{5}|}, & \text{for } 5 < T_{35} \leq 25 \\ v^{25} \ddot{a}^{(12)}_{\overline{K^{(12)}_{35} + (1/12) - 25}|} - P \bar{a}_{\overline{5}|}, & \text{for } T_{35} > 25 \end{cases}$$

(d) For a policy still in force at duration 5, the policy value can be derived using

$$\text{APV(FP}_5) = P \bar{a}_{35:\overline{5}|}$$

and

$$\text{APV(FB}_5) = 5 \bar{A}^{\;1}_{35:\overline{5}|} + P \big(\bar{I}\bar{A}\big)^{\;1}_{35:\overline{5}|} + 10P\, {}_{5}E_{35}\, \bar{A}^{\;1}_{40:\overline{20}|} + {}_{25}E_{35}\, \ddot{a}^{(12)}_{60}.$$

The policy value at duration 5 is therefore

$${}_{5}V = 5 \bar{A}^{\;1}_{35:\overline{5}|} + P \big(\bar{I}\bar{A}\big)^{\;1}_{35:\overline{5}|} + 10P\, {}_{5}E_{35}\, \bar{A}^{\;1}_{40:\overline{20}|} + {}_{25}E_{35}\, \ddot{a}^{(12)}_{60} - P \bar{a}_{35:\overline{5}|}.$$