Exercise 7.4

(a) With a starting policy value of ${}_0V = 0$, we use the recursive formula to calculate the next year's policy value:

$$({}_0V + P)(1+i) = {}_1V + q_{[50]}(1000 + {}_1V - {}_1V).$$

This simplifies to

$${}_1V = P(1+i) - 1000q_{[50]}$$

Following the same development, the following year's policy value can be determined based on

$$({}_1V + P)(1+i) = {}_2V + q_{[50]+1}(1000 + {}_2V - {}_2V)$$

which simplifies to

$$\begin{aligned} {}_2V &= ({}_1V + P)(1+i) - 1000q_{[50]+1} \\ &= (P(1+i) - 1000q_{[50]} + P)(1+i) - 1000q_{[50]+1} \\ &= P[(1+i)^2 + (1+i)] - 1000[q_{[50]}(1+i) + q_{[50]+1}] \end{aligned}$$

After 2 years, the death benefits are level at 20000 and at that time, the select period has expired so that the second year policy value can also be calculated using

$${}_2V = \text{APV(FB)} - \text{APV(FP)} = 20000A_{52} - P\ddot{a}_{52}.$$

Equating the two reserve equations, this leads us to a solution of P:

$$P = \frac{20000A_{52} + 1000[q_{[50]}(1+i) + q_{[50]+1}]}{\ddot{a}_{52} + [(1+i)^2 + (1+i)]}.$$

We can verify the following values based on the Standard Select Survival Model but with $i = 6\%$:

$$\ddot{a}_{52} = 14.87918 \text{ and } A_{52} = 0.1577825$$

and

$$q_{[50]} = 0.001033293 \text{ and } q_{[50]+1} = 0.001264444$$

and therefore, by substituting these values, we have

$$P = \frac{20000(0.1577825) + 1000[0.001033293(1.06) + 0.001264444]}{14.87918 + (1.06^2 + 1.06)} = 185.0818.$$

The policy value at the end of the second year is therefore

$${}_2V = 20000(0.1577825) - 185.0818(14.87918) = 401.7849.$$

(b) The policy value at $t = 2.25$, a period between valuation dates, can be exactly calculated again using the principle of recursion as

$$_{2.25}V = 20000 \times v^{0.75} \ _{.75}q_{52.25} + v^{0.75} \ _{.75}p_{52.25} \times \ _3V.$$

It can be verified that

$$_3V = 20000A_{53} - P\ddot{a}_{53} = 20000(0.1660246) - 185.0818(14.73357) = 593.576$$

and that

$$_{.75}p_{52.25} = \exp\left[-A(0.75) - \frac{B}{\log(c)}c^{52.25}(c^{0.75} - 1)\right] = 0.9988847,$$

where A, B, and c are the constants in the Standard Select survival model. By simple substitution of values, we finally arrive at

$$_{2.25}V = 20000 \times (1.06)^{-0.75}(1 - 0.9988847) + (1.06)^{-0.75}(0.9988847) \times 593.576 = 588.9127.$$

An approximation to this policy value at an interim date is one found at the bottom of page 206 of the text:

$$\begin{aligned} _{2.25}V &\approx (\,_2V + P) \times 0.75 + \,_3V \times 0.25 \\ &= (401.7849 + 185.0818) \times 0.75 + 593.576 \times 0.25 \\ &= 588.544. \end{aligned}$$