Exercise 7.11

(a) At issue, the APV of future premiums is

$$\text{APV}(\text{FP}_0) = G\ddot{a}_{[50]},$$

the APV of future benefits is

$$\text{APV}(\text{FB}_0) = 10000A_{[50]},$$

and the APV of future expenses is

$$\text{APV}(\text{FE}_0) = 0.17G + 0.05G\ddot{a}_{[50]} + 90 + 10\ddot{a}_{[50]}.$$

By the equivalence principle, we have

$$\text{APV}(\text{FP}_0) = \text{APV}(\text{FB}_0) + \text{APV}(\text{FE}_0)$$

and the gross annual premium, G, is therefore

$$G = \frac{10000A_{[50]} + 90 + 10\ddot{a}_{[50]}}{0.95\ddot{a}_{[50]} - 0.17}.$$

Based on the Standard Select Survival Model with $i = 4.5\%$, one can verify that

$$\ddot{a}_{[50]} = 18.12588$$

and that

$$A_{[50]} = 1 - (.045/1.045)(18.12588) = 0.2194596$$

so that we have

$$G = \frac{10000(0.2194596) + 90 + 10(18.12588)}{0.95(18.12588) - 0.17} = 144.6284.$$

(b) The policy value at each year end can be evaluated recursively. Starting with ${}_0V = 0$, we have

$${}_1V = \frac{({}_0V + 0.78G - 100)(1.045) - 10000q_{[50]}}{1 - q_{[50]}} = 3.056835$$

where $q_{[50]} = 0.001033293$ based on the Standard Select Model. For $k \geq 2$, we have

$${}_kV = \frac{({}_{k-1}V + 0.95G - 10)(1.045) - 10000q_{[50]+k-1}}{1 - q_{[50]+k-1}}.$$

The policy values, applying the recursion above, for the first few years are summarized in the table below:

k	${}_kV$	k	${}_kV$	k	${}_kV$
0	0.000	10	1241.768	20	3027.746
1	3.057	11	1401.558	21	3226.591
2	123.836	12	1565.778	22	3428.225
3	248.216	13	1734.375	23	3632.331
4	376.893	14	1907.272	24	3838.566
5	509.926	15	2084.371	25	4046.555
6	647.365	16	2265.551	26	4255.898
7	789.248	17	2450.665	27	4466.168
8	935.603	18	2639.540	28	4676.915
9	1086.444	19	2831.977	29	4887.667

(c) Since the only source of profit is from the higher than assumed interest rate in each year, we can compute the bonus at the end of year k, denoted by bonus_k, recursively as follows:

$$\text{bonus}_1 = 0.90 \times \frac{({}_0V + 0.78G - 100)(0.055 - 0.045)}{1 - q_{[50]}} = 0.1154106$$

and for $k \geq 2$, we have

$$\text{bonus}_k = 0.90 \times \frac{({}_{k-1}V + 0.95G - 10)(0.055 - 0.045)}{1 - q_{[50]+k-1}}.$$

These values for the first few years are summarized below:

k	bonus_k	k	bonus_k	k	bonus_k
1	0.115	11	12.364	21	28.695
2	1.176	12	13.813	22	30.542
3	2.264	13	15.303	23	32.425
4	3.386	14	16.836	24	34.341
5	4.547	15	18.409	25	36.290
6	5.747	16	20.024	26	38.271
7	6.988	17	21.680	27	40.282
8	8.270	18	23.376	28	42.323
9	9.593	19	25.111	29	44.392
10	10.958	20	26.884	30	46.490

(d) The profit in year k, denoted by profit_k, can be recursively computed based on

$$\text{profit}_1 = \frac{({}_0V + 0.78G - 100)(0.055 - 0.045)}{1 - q_{[50]}} = 0.1154106$$

and for $k \geq 2$, we have

$$\text{profit}_k = \frac{({}_{k-1}V + 0.95G - 10)(0.055 - 0.045)}{1 - q_{[50]+k-1}}.$$

These profit values are prior to the distribution of the 90% dividends to the policyholders. While it is not clear from the problem whether the expected present value required is for profits prior to or after dividends, we give both values. For profits to the insurer prior to the distribution of dividends, the expected present value is given by

$$\text{EPV(profits)} = \sum_k v^k \times {}_{k-1}p_{[50]} \times \text{profit}_k = 301.7388.$$

The expected present value is given by

$$0.10 \times 301.7388 = 30.17388$$

after the 90% dividends distributed. Neither of this matches the given solution in the book because it is based on the following formula:

$$\sum_k v^k \times {}_{k-1}p_{[50]} \times q_{[50]+k} \times \text{bonus}_k = 263.3683.$$

This computation clearly does not make sense in two respect: (i) it considers the bonus (and not the profit or remaining profit after bonus), and (ii) it does not consider value on a per policy basis.

(e) It is too early to be able to recoup a large proportion of the initial expense, for surrenders at the end of the first year. Hence, a reasonable surrender benefit is nothing.