Exercise 6.18

With the extra risk, we have

$$ {}_tp'_x = \exp\left[-\int_0^t \mu'_{x+s}ds\right] = \exp\left[-\int_0^t \big(\mu_{x+s}+\phi\big)ds\right] = \exp\left[-\int_0^t \mu_{x+s}ds\right]\cdot \mathrm{e}^{-\phi t} = {}_tp_x \cdot \mathrm{e}^{-\phi t} $$

so that we have

$$ \begin{aligned} \bar{A}'_x &= \int_0^\infty v^t\, {}_tp'_x \mu'_{x+t}dt \\ &= \int_0^\infty v^t\, {}_tp_x \cdot \mathrm{e}^{-\phi t}\big(\mu_{x+t}+\phi\big)dt \\ &= \int_0^\infty \mathrm{e}^{-(\delta+\phi)t}\, {}_tp_x \mu_{x+t}dt + \phi \int_0^\infty \mathrm{e}^{-(\delta+\phi)t}\, {}_tp_x dt \\ &= \bar{A}^j_x + \phi \bar{a}^j_x \end{aligned} $$

where $\bar{A}^j_x$ and $\phi\bar{a}^j_x$ are respectively, continuous whole life insurance and whole life annuity evaluated at the force of interest

$$ \delta' = \delta + \phi. $$

Equivalently, this leads to an annual effective interest rate of

$$ j = (1+i)\mathrm{e}^{\phi} - 1. $$