Reserves
liability
Multiple state
lives
decrements

alive — cancer, HD, other

MW 1:50-2:55
F 1:50-2:55
Wed

1st 6 weeks except Jan 28

Premium Calculation - continued

STT 455

Ch. 1

MLC

Lecture: Weeks 1-2

7, 8, 9, 10, 12, 13

11

1 HW
1 CT
2 CT

UL

tidbits
pension
emerging costs

1 life
2 decrement

states
live
dead
ill

hired — ill — die
retire

Recall some preliminaries

An insurance policy (life insurance or life annuity) is funded by contract premiums:

- once (single premium) made usually at time of policy issue, or
- a series of payments (usually contingent on survival of policyholder) with first payment made at policy issue
- to cover for the benefits, expenses associated with initiating/maintaining contract, profit margins, and deviations due to adverse experience.

Net premiums (or sometimes called benefit premiums)

- considers only the benefits provided
- nothing allocated to pay for expenses, profit or contingency margins

Gross premiums (or sometimes called expense-loaded premiums)

- covers the benefits and includes expenses, profits, and contingency margins

Chapter summary - continued

$E(L) = 0$

- Present value of future loss (negative profit) random variable
- Premium principles
  - the equivalence principle (or actuarial equivalence principle)
  - portfolio percentile premiums
- Return of premium policies
- Substandard risks Selection
- Chapter 6 (sections 6.8, 6.9) of Dickson, et al.

Net random future loss

- An insurance contract is an agreement between two parties:
  - the insurer agrees to pay for insurance benefits;
  - in exchange for insurance premiums to be paid by the insured.
- The insurer's net random future loss is defined by

$$L_0^n = \mathsf{PVFB}_0 - \mathsf{PVFP}_0.$$

$+ \mathrm{PVFE}_0$

  where PVFB_0 is the present value, at time of issue, of future benefits to be paid by the insurer

  and

  PVFP_0 is the present value, at time of issue, of future premiums to be paid by the insured.

- Note: this is also called the present value of future loss random variable (in the book), and if no confusion, we may simply write this as L_0.

The principle of equivalence

$E[\text{PVFB}_0 - \text{PVFP}_0] = 0$

- The net premium, generically denoted by P, may be determined according to the principle of equivalence by setting

$$\mathsf{E}\left[L_0^n\right] = 0.$$

- The expected value of the insurer's net random future loss is zero.
- This is then equivalent to setting $\mathsf{E}\left[\text{PVFB}_0\right] = \mathsf{E}\left[\text{PVFP}_0\right]$. In other words, at issue, we have

$$\text{APV}(\underbrace{\text{Future Premiums}}_{FP}) = \text{APV}(\underbrace{\text{Future Benefits}}_{FB}).$$

FE

Gross premium calculations

- Treat expenses as if they are a part of benefits. The gross random future loss at issue is defined by

$$L_0^g = \mathsf{PVFB}_0 + \mathsf{PVFE}_0 - \mathsf{PVFP}_0,$$

P G π

where PVFE_0 is the present value random variable associated with future expenses incurred by the insurer.

- The gross premium, generically denoted by G, may be determined according to the principle of equivalence by setting

$$\mathsf{E}\left[L_0^g\right] = 0.$$

- This is equivalent to setting $\mathsf{E}\left[\mathsf{PVFB}_0\right] + \mathsf{E}\left[\mathsf{PVFE}_0\right] = \mathsf{E}\left[\mathsf{PVFP}_0\right]$. In other words, at issue, we have

$$\mathsf{APV}(\mathsf{FP}_0) = \mathsf{APV}(\mathsf{FB}_0) + \mathsf{APV}(\mathsf{FE}_0).$$

fully discrete whole life of $\underline{1000}$ on (45)

$\Downarrow$

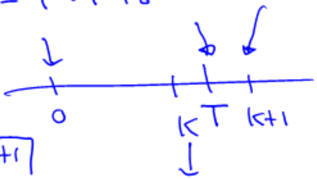
benefit e.o.y. death

premium b.o.y.

expenses are 30 at issue & then 5 per year thereafter.

Gross premium = ? = G $\Rightarrow$

$$L_0 = PVFB_0 + PVFE_0 - PVFP_0$$



$$1000v^{K+1} + 25 + 5\ddot{a}_{\overline{K+1|}} - G\ddot{a}_{\overline{K+1|}}$$

$$E[L_0]=0 \Rightarrow \underbrace{APV(FP)} = APV(FB) + APV(FE)$$

$$G\cancel{\ddot{a}_{45}} = \frac{1000A_{45} + 25 + 5\ddot{a}_{45}}{\ddot{a}_{45}}$$

Portfolio percentile premium ~~principle~~ group

Suppose insurer issues a portfolio of N "identical" and "independent" policies where the PV of loss-at-issue for the i-th policy is $L_{0,i}$.

$\Downarrow$

true

The total portfolio (aggregate) future loss is then defined by

$$L_{\text{agg}} = L_{0,1} + L_{0,2} + \cdots + L_{0,N} = \sum_{i=1}^{N} L_{0,i}$$

N very large

$\sim$ Normally distribution

Its expected value is therefore

$$\mathsf{E}[L_{\text{agg}}] = \sum_{i=1}^{N} \mathsf{E}\left[L_{0,i}\right]$$

and, by "independence", the variance is

$$\mathrm{Var}[L_{\text{agg}}] = \sum_{i=1}^{N} \mathrm{Var}\left[L_{0,i}\right].$$

independent

Portfolio percentile premium principle

The portfolio percentile premium principle sets the premium P so that there is a probability, say α with $0 < \alpha < 1$, of a positive gain from the portfolio.

In other words, we set P so that

$$\Pr[L_{\text{agg}} < 0] = \alpha.$$

large $\alpha = 1\%$ 99%
2% 98%
5% 95%

Note that loss could include expenses.

$$\Pr\left[\underbrace{\frac{L_{agg} - E[L_{agg}]}{\sqrt{Var[L_{agg}]}}}_{\approx Z \to N(0,1)} > \frac{-E[L_{agg}]}{\sqrt{Var[L_{agg}]}}\right] = 1-\alpha$$

$$\Pr[L_{agg} > 0] = \underbrace{1-\alpha}_{\text{small}}$$

N polcies "identical" "independent"

L_{agg} = aggregate loss

$L_{0,i}$ = PV of loss per policyholder

issue, policyholder

$$= \sum_{i=1}^{N} L_{0,i} \Rightarrow Pr[L_{agg} > 0] = \underbrace{1-\alpha}_{\text{small}}$$

5%

$$Pr\left[\underbrace{\frac{\sum L_{0,i} - E[L_{agg}]}{\sqrt{Var[L_{agg}]}}}_{\text{CLT}} > \frac{-E[L_{agg}]}{\sqrt{Var[L_{agg}]}}\right]$$

Central Limit Theorem

$\approx Z \sim N(0,1)$

$\alpha = 95\%$, $= 1.645$

Solve for P ✓

fully discrete whole life to (x) benefit = $\underline{1000}$

$i = 5\%$

$d = \frac{i}{1+i}$,

no expenses

$A_x = 0.20$

${}^2A_x = 0.06$ A_x @ 2δ

500 policies all identical, independent lifetimes

P = premium per dollar

Premium so that $P_r(L_{agg} \leq 0) = 0.95$

$$L_{0,i} = 1000 v^{K+1} - 1000 P \ddot{a}_{\overline{K+1}|} \qquad \ddot{a}_{\overline{K+1}|} = \frac{1-v^{K+1}}{d}$$

$E[v^{K+1}] = A_x$

$$= 1000 \left(1 + \frac{P}{d}\right) v^{K+1} - 1000 \frac{P}{d}$$

$$E[L_{0,i}] = 1000\left(1+\frac{P}{d}\right)(.20) - 1000\, P/d = 200 - 800\, P/d$$

$$Var[L_{0,i}] = 1000^2\left(1+\frac{P}{d}\right)^2 [\underbrace{{}^2A_x - A_x^2}_{.02}] = \underline{(1000)^2\left(1+\frac{P}{d}\right)^2 (.02)}$$

$$E[L_{agg}] = 500\left(200 - 800\,P/d\right)$$

$$Var[L_{agg}] = 500\,(1000)^2\,(1+P/d)^2\,(.02)$$

$$Pr[L_{agg} < 0] = .95$$

$$Pr\left(Z < \underbrace{\frac{0 - \qquad}{\sqrt{\qquad}}}_{1.645}\right) = .95$$

$$\frac{-\sqrt{500}\,\cancel{-500}\left(200-800\,P/d\right)}{\sqrt{\cancel{500}\,(1000)^2\,(1+P/d)^2(.02)}} = 1.645$$

$\Rightarrow$ <u>solve for P!</u> ✓

1:50 – 2:55pm A108 -

MWF $\rightarrow$

$$-\sqrt{500}\left(.2-.8\frac{P}{d}\right) = 1.645\sqrt{.02}\left(1+\frac{P}{d}\right) \qquad d=\frac{.05}{1.05}$$

Solve for P: $\quad P = .01268906 \ast 1000 = \underline{\underline{12.68906}}$

Compare with equivalence: $\quad P = \dfrac{1000\,A_x}{\ddot{a}_x} = \dfrac{1000\,A_x^{\,.2}}{\dfrac{1-A_x}{d}}, \quad d = \dfrac{.05}{1.05}, \quad = \underline{\underline{11.90476}}$

Example 6.12

$i = 5\%$

$i^{(12)} = 12\left[1.05^{1/12} - 1\right]$

$d^{(12)} = 12\left[1 - 1.05^{-1/12}\right]$

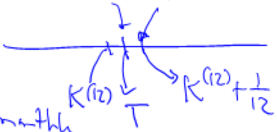
Consider Example 6.12.

(a) equivalence principle

(b) $\Pr[L_{0} > 0] = .05$

whole life to [30]

benefit = 100,000 payable at end of month of death



premium is monthly

initial expenses: 15% of 1st yr premium

renewal expenses: 4% of every premium including the 1st year

$K^{(12)}$ = monthly curtate lifetime

(a)

$P =$ monthly premium

$12P =$ annualized premium

each policy loss at issue

$$L_{0,i} = 100{,}000 \, \underbrace{v^{K^{(12)}+\frac{1}{12}}} + \underbrace{0.15 * 12P} + 0.04 * 12P * \underbrace{\ddot{a}^{(12)}_{\overline{K^{(12)}+\frac{1}{12}}|}}_{1-v}$$

$$- 12P \, \ddot{a}^{(12)}_{\overline{K^{(12)}+\frac{1}{12}}|} \quad 1-v$$

$$E[L_{0,i}] = \overbrace{100{,}000 \, A^{(12)}_{[30]}} + 1.8P - \overbrace{.96(12)}^{11.52} P \, \ddot{a}^{(12)}_{[30]} = 0$$

$$P = \frac{100{,}000 \, A^{(12)}_{[30]}}{11.52 \, \ddot{a}^{(12)}_{[30]} - 1.8}$$

$$\boxed{P = 36.3946}$$

$A_{[30]} = .07693$

$$A^{(12)}_{[30]} \cong \frac{i}{i^{(12)}} A_{[30]} = .07867745$$

$$\ddot{a}^{(12)}_{[30]} = \frac{1 - A^{(12)}_{[30]}}{d^{(12)}} = 18.92178$$

$$\ddot{a}^{(12)}_{\overline{n}|} = \frac{1 - v^n}{d^{(12)}}$$

10,000

$$E[L_{0,i}] = 7866.14 - 216.18P$$

$$Var[L_{0,i}] = \left[100{,}000 + \frac{.96(12)P}{d^{(12)}}\right]^2 \underbrace{Var\left[v^{K^{(12)}+\frac{1}{12}}\right]}_{\left[{}^2A^{(12)}_{[30]} - \left(A^{(12)}_{[30]}\right)^2\right]}$$

$$i^* \qquad {}^2A_{[30]}$$

$$i^{(12)*}$$

$$\underbrace{(100{,}000 + 236.59P)^2}_{} \qquad .0053515$$

$$\Pr[L_{agg} > 0] \cong \Pr\left[Z > \frac{-10{,}000(2866.14 - 216.18P)}{\sqrt{(100{,}000 + 236.59P)^2(.0053515)(10{,}000)}}\right]$$

$$\Rightarrow \; P = 36.99$$

Illustrative example 1

An insurer sells 100 fully discrete whole life insurance policies of $1, each of the same age 45. You are given:

- All policies have independent future lifetimes.
- $i = 5\%$
- $\ddot{a}_{45} = 17.81876$
- ${}^2A_{45} = .03450$

Using the Normal approximation:

1. Calculate the annual contract premium according to the portfolio percentile premium principle with $\alpha = 0.95$.
2. Suppose the annual contract premium is set at 0.01 per policy. Determine the smallest number of policies to be sold so that the insurer has at least a 95% probability of a gain from this portfolio of policies.

100 policies

$$L_{0,i} = v^{K+1} - P\,\ddot{a}_{\overline{K+1|}} = \underbrace{\left(1+\frac{P}{d}\right)v^{K+1} - \frac{P}{d}}$$

$\ddot{a}_{\overline{K+1|}} = \dfrac{1-v^{K+1}}{d}$

$$E[L_{0,i}] = \left(1+\frac{P}{d}\right)A_{45} - \frac{P}{d}$$

$$\ddot{a}_{45} = 17.81876$$

$$A_{45} = 1 - d\,\ddot{a}_{45} = \underbrace{1 - \frac{.05}{1.05}\,17.81876}_{.1514876}$$

$$Var[L_{0,i}] = \left(1+\frac{P}{d}\right)^2\left[{}^2A_{45} - (A_{45})^2\right]$$

$$\underbrace{.0345 - (.1514876)^2}$$

$$= \qquad .01155150$$

$$Pr[L_{agg} > 0] = Pr\left[Z > \frac{0 - 100\left[\left(1+\frac{P}{d}\right)(.1514876) - P/d\right]}{\sqrt{.01155150\left(1+\frac{P}{d}\right)^2(100)}}\right] \Rightarrow \underline{P = .00969585\text{2}8}$$

$N = ?$ $\quad\quad p = .01$ ✓

$$E[L_{0,i}] = \left(1 + \frac{.01}{.05/1.05}\right)(.1514876) - \frac{.01}{.05/1.05} = a$$

$$Var[L_{0,i}] = \left(1 + \frac{.01}{.05/1.05}\right)^2 (.01155150) = b$$

$$\Pr[L_{agg} > 0] \approx \Pr\left[Z > \underbrace{\frac{0 - aN}{\sqrt{bN}}}\right] = 0.05$$

$$\frac{-a\cancel{N}^{\sqrt{N}}}{\sqrt{b\cancel{N}}} = 1.645 \Rightarrow (\sqrt{N})^2 = \left(\frac{1.645\sqrt{b}}{-a}\right)^2$$

$$N = 64.19764$$

Choose $N = 65$ policies ✓

Return of premium policies

Consider a fully discrete whole life insurance to (x) with benefit equal to $\$B$ plus return of all premiums accumulated with interest at rate j.

The net random future loss in this case can be expressed as

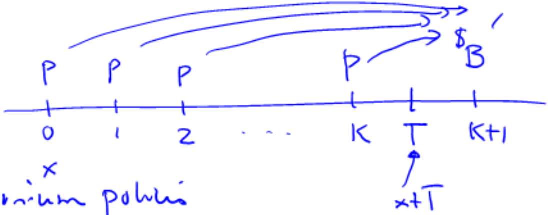
$$L_0 = P\, \ddot{s}_{\overline{K+1}|j}\, v^{K+1} + B\, v^{K+1} - P\, \ddot{a}_{\overline{K+1}|},$$

for $K = 0, 1, \ldots$ and $\ddot{s}_{\overline{K+1}|j}$ is calculated at rate j. All other actuarial functions are calculated at rate i.

Consider the following cases:

- Let $j = 0$. This implies $\ddot{s}_{\overline{K+1}|j} = (K+1)$ and the annual benefit premium will be
$$P = \frac{B\, A_x}{\ddot{a}_x - (IA)_x}.$$

$\underbrace{\text{fully discrete}}_{\Downarrow}$ whole life \$B to (x) no expenses



return of premium policy

premiums accumulate at j

premium are calculated at i

+ PVFE

$$\underbrace{\text{Loss at issue}}_{L_0} = PVFB - PVFP$$

$$L_0 = Bv^{K+1} + P\,\ddot{s}_{\overline{K+1}|j}\,v^{K+1} - P\,\ddot{a}_{\overline{K+1}|}$$

Case 1: $j=0$ $\quad L_0 = \underbrace{Bv^{K+1}} + \underbrace{P(K+1)\,v^{K+1}} - \underbrace{P\,\ddot{a}_{\overline{K+1}|}}$

$$E[L_0] = 0 \Rightarrow \underbrace{B\,A_x + P\,(IA)_x - P\,\ddot{a}_x = 0}_{P = \dfrac{B A_x}{\ddot{a}_x - (IA)_x}}$$

Case 2: $j = i$

$$L_0 = B v^{k+1} + \cancel{P\,\underbrace{\ddot{s}_{\overline{k+1|}}\, v^{k+1}}_{\ddot{a}_{\overline{k+1|}}}} - \cancel{P\,\ddot{a}_{\overline{k+1|}}} > 0$$

no premium possible

Case 3: $j > i \Rightarrow$ no possible premium

$$L_0 = B v^{k+1} + P\,\ddot{s}_{\overline{k+1|}\,j}\, v^{k+1} - P\,\ddot{s}_{\overline{k+1|}}\, v^{k+1}$$

$$= \underbrace{Bv^{k+1}}_{>0} + \underbrace{Pv^{k+1}}_{>0}\underbrace{\left(\ddot{s}_{\overline{k+1}|j} - \ddot{s}_{\overline{k+1}|i}\right)}_{>0}$$

$L_0 > 0 \implies$ no possible P!

Case 4: $j < i$

$$L_0 = Bv^{k+1} + P\,\ddot{s}_{\overline{k+1}|j}\,v^{k+1} - P\,\ddot{a}_{\overline{k+1}|}$$

$$\ddot{s}_{\overline{k+1}|j}\,v^{k+1} = \frac{(1+j)^{k+1} - 1}{d_j}$$

$$= Bv^{k+1} + P\frac{\left[v_{j^*}^{k+1} - v^{k+1}\right]}{d_j} - P\,\ddot{a}_{\overline{k+1}|}$$

$$E[L_0] = B A_x + \frac{P}{d_j}\left[A_{x@j^*} - A_x \right] - P\ddot{a}_x' = 0$$

$$\boxed{P = \frac{B A_x}{\ddot{a}_x + \frac{1}{d_j}\left(A_x - A_{x@j^*} \right)}}$$

$$V_{j^*}^{\cancel{k+1}} = (1+j)^{\cancel{k+1}} v^{\cancel{k+1}} = \frac{1+j}{1+i}$$

$$\frac{1}{1+j^*} \Rightarrow j^* = \frac{1+i}{1+j} - 1 > 0$$

Return of Premise

1. $j=0 \Rightarrow$ possible
2. $j=i \Rightarrow$ not possible
3. $j>i \Rightarrow$ not possible
4. $j<i \Rightarrow$ possible

- continued

- Let $i = j$. In this case, the loss $L_0 = B\,v^{K+1}$ because $\ddot{s}_{\overline{K+1}|j}\,v^{K+1} = \ddot{a}_{\overline{K+1}|}$. Thus, there is no possible premium because all premiums are returned and yet there is an additional benefit of $\$B$.

- Let $i < j$. Then we have

$$L_0 = P\left(\ddot{s}_{\overline{K+1}|j} - \ddot{s}_{\overline{K+1}|}\right)v^{K+1} + B\,v^{K+1},$$

  which is always positive because $\ddot{s}_{\overline{K+1}|j} > \ddot{s}_{\overline{K+1}|}$ when $i < j$. No possible premium.

- continued

- Let $i > j$. Then we can write the loss as

$$L_0 = P\,\frac{v_{j^*}^{K+1} - v^{K+1}}{d_j} + B\,v^{K+1} - P\,\ddot{a}_{\overline{K+1}|}$$

where $d_j = 1 - [1/(1+j)]$ and v_{j^*} is the corresponding discount rate associated with interest rate $j^* = [(1+i)/(1+j)] - 1$. Here,

$$P = \frac{A_x}{\ddot{a}_x - \frac{(A_x)_{j^*} - A_x}{d_j}},$$

where $(A_x)_{j^*}$ is a (discrete) whole life insurance to (x) evaluated at interest rate j^*.

Illustrative example 2

$d = \frac{i}{1+i}$

$= \frac{.04}{1.04}$

For a whole life insurance on (40), you are given:

- Death benefit, payable at the end of the year of death, is equal to \$10,000 plus the return of all premiums paid without interest. $j=0$
- $B=$ Annual benefit premium of 290.84 is payable at the beginning of each year.
- $(IA)_{40} = 8.6179$
- $i = 4\%$

Calculate $\ddot{a}_{40}$.

$$P = \frac{BA_x}{\ddot{a}_x - (IA)_x}$$

$1-d\ddot{a}_{40}$

$$290.84 = \frac{10{,}000 A_{40}}{\ddot{a}_{40} - 8.6179}$$

$$290.84\left(\ddot{a}_{40} - 8.6179\right) = 10{,}000\left(1 - \frac{.04}{1.04}\ddot{a}_{40}\right)$$

$$\ddot{a}_{40}\left(290.84 + 10{,}000\left(\frac{.04}{1.04}\right)\right) = 10{,}000 + 290.84(8.6179)$$

$$\ddot{a}_{40} = 18.51555$$

SOA Question #22

Consider Question #22 from Fall 2012 SOA MLC Exam:

You are given the following information about a special fully discrete 2-payment, 2-year term insurance on (80):

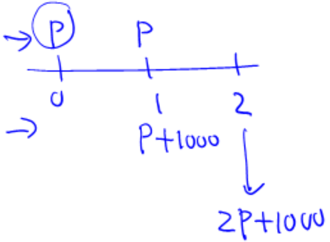
- Mortality follows the `Illustrative Life Table`.
- $i = 0.0175$
- The death benefit is 1000 plus a return of all premiums paid without interest.

  P
- Level premiums are calculated using the equivalence principle.

Calculate the benefit premium for this special insurance.

For practice: try calculating the benefit premium if the return of all premiums paid comes with an interest of say 0.01.

$$APV(FP) = APV(FB)$$



$$P + P v p_{80} = (P+1000) v q'_{80} + (2P+1000) v^2 p_{80} q_{81}$$

$$P\left(\underbrace{1 + v p_{80} - v q'_{80} - 2v^2 p_{80}}\, q_{81}\right) = \underline{1000\left(v q_{80} + v^2 p_{80} q'_{81}\right)}$$

$$P = \frac{1000\left(\frac{1}{1.0175} 0.08030 + \frac{1}{1.0175^2} 0.91970 (0.08764)\right)}{\underbrace{1 + \frac{1}{1.0175} 0.91970} - \cdots\cdots}$$

$$P = \frac{156.77}{1.90388 - 0.23463} = \underline{\underline{\underline{93.62}}}$$

Pricing with extra or substandard risks

An impaired individual, or one who suffers from a medical condition, may still be offered an insurance policy but at a rate higher than that of a standard risk.

Generally there are three possible approaches:

- age rating: calculate the premium with the individual at an older age
- constant addition to the force of mortality: $\mu^s_{x+t} = \mu_{x+t} + \phi$, for $\phi > 0$
- constant multiple of mortality rates: $q^s_{x+t} = \min(cq_{x+t}, 1)$, for $c > 1$

Read Section 6.9.

life annuity due

single premium

3 year term

to (40)

$i = 6\%$ $\qquad \mu^*$

$$\mu^S = \mu^{ILT} + .001$$

ILT = Illustrative Life Table

$$\ddot{a}^S_{40:\overline{3}|} = 1 + v\,p^S_{40} + v^2\,p^S_{40}\,p^S_{41}$$

$$p^S_{40} = e^{-\mu^S_{40}} = e^{-\mu^{ILT}_{40}}\,e^{-.001}$$

$$v = \frac{1}{1.06}$$

$$p^{ILT}_{40} = 1 - \frac{2.78}{1000}$$

$$p^{ILT}_{41} = 1 - \frac{2.98}{1000}$$

$$= 1 + v\,p^{ILT}_{40}\,e^{-.001} + v^2\,p^{ILT}_{40}\,p^{ILT}_{41}\left(e^{-.001}\right)^2 = \underline{\underline{2.822943}}$$

$$\ddot{a}_{40:\overline{3}|} = 1 + v\,p^{ILT}_{40} + v^2\,p^{ILT}_{40}\,p^{ILT}_{41} = \underline{\underline{2.825651}}$$

Published SOA question #45

age rating-

Your company is competing to sell a life annuity-due with an APV of \$500,000 to a 50-year-old individual.

Based on your company's experience, typical 50-year-old annuitants have a complete life expectancy of 25 years. However, this individual is not as healthy as your company's typical annuitant, and your medical experts estimate that his complete life expectancy is only 15 years.

You decide to price the benefit using the issue age that produces a complete life expectancy of 15 years. You also assume:

- For typical annuitants of all ages, $\underbrace{\ell_x = 100(\omega - x)}_{\text{linear in } x}$, for $0 \leq x \leq \omega$.
- $i = 0.06$

linear in x ⟹ Uniform ⟹ DeMoivre

Calculate the annual benefit premium that your company can offer to this individual.

$\underset{\sim}{APV} = \cancel{500{,}000}\ B\ \ddot{a}_{\cancel{50}\ 70}$

$500{,}000 = B$

Uniform $l_x = 100(\omega - x)$

$T_x \sim (0, \omega - x)$

$E[T_x] = \dfrac{\omega - x}{2}$

$x = 50 \Rightarrow \dfrac{\omega - 50}{2} = 25$

$\omega = 100$

$T_x \sim (0, 100 - x)$

$E[T_x] = \dfrac{100 - x}{2} = 15$

$\underline{\underline{x = 70}}$

$$\ddot{a}_x = \sum_{t=0}^{\infty} v^t\,{}_tp_x$$

$$\ddot{a}_{70} = \sum_{t=0}^{\cancel{30}\ 29} v^t\,\underbrace{{}_tp_{70}}_{\frac{l_{70+t}}{l_{70}}}$$

$$= \sum_{t=0}^{29} v^t\left(1 - t/30\right)$$

$$A_{70} = \sum_{t=0}^{29} v^{t+1}\frac{d_{70+t}}{l_{70}} \qquad d_{70+t} \to \frac{\cancel{100}}{} \qquad l_{70} \to \cancel{100}(100-70)$$

$\cancel{A_x}$

$d_x = l_x - l_{x+1}$

$d_{70+t} = l_{70+t} - l_{71+t}$

$= 100(30 - \cancel{t} - 29 + \cancel{t})$

$= 100$

$$= \frac{1}{30}\sum_{t=0}^{29} v^{t+1} = \frac{1}{30}\Big(\underbrace{v + v^2 + \dots + v^{30}}_{13.76483}\Big)$$

0.45882277

$$\ddot{a}_{70} = \frac{1 - A_{30}}{d} = \frac{1 - \frac{1}{30}(13.76483)}{.06/1.06} = \underline{9.560711}$$ ok

$$B = \frac{500{,}000}{\ddot{a}_{70}} = \frac{500{,}000}{9.560711} = 52{,}297.37$$

please verify

Constant from A or $\ddot{a}$

Other terminologies and notations used

Expression	Other terms/symbols used
net random future loss	loss at issue
L_0	${}_0L$
net premium	benefit premium
gross premium	expense-loaded premium
equivalence principle	actuarial equivalence principle
generic premium	G P π
substandard	may be superscripted with * or s