STT 456 Review Problems for Class Test 2
Monday, April 6, 2015

1. An automobile insurance company classifies drivers according to various states:

   State 1: Excellent

   State 2: Good

   State 3: Bad

   State 4: Terrible and has to be discontinued

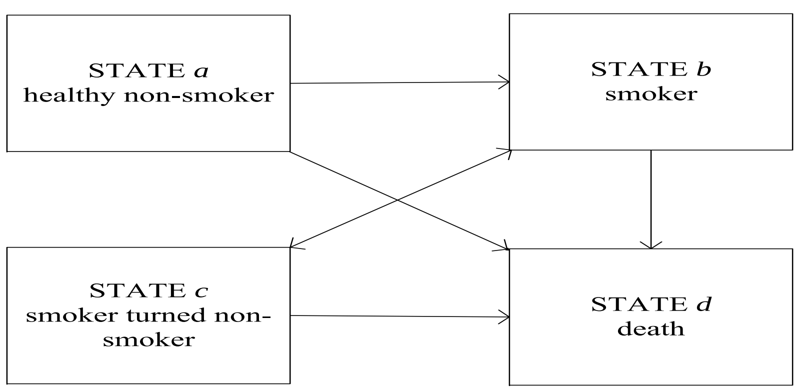
   Assume transitions follow a time-homogeneous Markov Chain model with the following transition matrix:

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{pmatrix}
0.8 & 0.1 & 0.1 & 0.0 \\
0.2 & 0.5 & 0.2 & 0.1 \\
0.0 & 0.1 & 0.6 & 0.3 \\
0.0 & 0.0 & 0.0 & 1.0
\end{pmatrix}
\end{array}
$$

   At the start of year 1, ten new drivers are insured and classified as Excellent drivers.

   Calculate the probability that during the first 3 years, half of these new drivers become Terrible drivers and therefore have to be discontinued.

2. An insurance company determines its policy premiums according to a multiple state model, shown in the figure below, with states: (a) healthy non-smoker (who has never smoked before), (b) smoker, (c) smoker turned non-smoker, and (d) death.



   The homogeneous transition probability matrix is given by:

$$
\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c}
\begin{array}{cccc} a & b & c & d \end{array} \\
\begin{pmatrix}
0.8 & 0.1 & 0.0 & 0.1 \\
0.0 & 0.4 & 0.2 & 0.4 \\
0.0 & 0.1 & 0.6 & 0.3 \\
0.0 & 0.0 & 0.0 & 1.0
\end{pmatrix}
\end{array}.
$$

The company issues a two-year term policy that provides death benefit of 1 at the end of the year of death. Assume transitions occur at the end of each year and that interest rate is $i = 10\%$.

Calculate the ratio of the actuarial present value of the policy for a healthy non-smoker to that of a smoker.

3. The `Illustrative Service Table` is a multiple decrement table based on four decrements: (d) is death, (w) is withdrawal from employment, (i) is sickness, and (r) is retirement. Table is attached.

   Using the table, calculate the ${}_{2|2}q_{60}^{(\mathrm{r})}$ and interpret this value.

4. A two-year fully discrete term insurance policy issued to (x) pays \$3,000 if death is due to accidental causes. You are given the following table in which decrement 1 represents accidental death and decrement 2 represents death from all other causes:

x	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
50	10,000	440	360
51	9,200	1,350	1,500
52	6,350	1,728	1,965

   Annual premiums are determined using the equivalence principle and a 5% annual effective interest rate.

   Calculate the annual premium payable for this policy.

5. For a husband and wife with ages x and y, respectively, you are given:

   - $\mu_{x+t} = 0.03$ for all $t > 0$
   - $\mu_{y+t} = 0.01$ for all $t > 0$
   - Their future lifetimes are independent.
   - A special insurance pays \$20 at the moment of the husband's death if he dies first and \$5 if he dies after his wife.
   - $\delta = 5\%$

   Calculate the actuarial present value of the benefits for this special insurance.

6. A fully discrete whole life insurance of \$ 150,000 is issued to (50). You are given:

   - The contract annual premium is \$ 3,620.
   - Expenses are incurred at the beginning of each year: the first year expense is \$ 1,650 and the second year expense is \$ 400.

- The interest rate earned in the first year is 5.5%, and 4.5% in the second year.
- Withdrawals occur at the end of the year. If policy withdraws in the first year, there is no cash value payable. If policy withdraws in the second year, the cash value payable is \$ 2,760.
- The applicable decrement table is given below (with w denoting withdrawal or lapse and d denoting death):

x	$q_x^{(d)}$	$q_x^{(w)}$
50	0.0059	0.0350
51	0.0064	0.0325

Calculate the policy's asset share at the end of two years.

7. You are given:

   - A husband and wife, with independent future lifetimes, are of the same age 45.
   - The husband's mortality follows a constant force with $\mu_{45+t} = 0.03$ for all $t \geq 0$.
   - The wife's mortality follows a de Moivre's law with $\omega = 100$.
   - $\delta = 6\%$

   Calculate $\overline{a}_{\overline{45:45}}$.

   Hint: $\displaystyle\int te^{-kt}dt = -\frac{1}{k}te^{-kt} - \frac{1}{k^2}e^{-kt}$, for any constant $k > 0$.

8. In a double decrement model, you are given:

   - Decrement 1 occurs only once in the middle of each year.
   - Decrement 2 is uniformly distributed in its associated single decrement table.
   - $q_x'^{(1)} = 0.02$
   - $q_x'^{(2)} = 0.04$

   Calculate $\dfrac{q_x^{(1)}}{q_x^{(\tau)}}$.

9. For (x) and (y) with independent future lifetimes, you are given:

   - $\bar{a}_x = 10.06$
   - $\bar{a}_y = 11.95$
   - $\bar{a}_{\overline{xy}} = 12.59$
   - $\bar{A}_{x\overset{1}{y}} = 0.09$
   - $\delta = 0.07$

   Calculate $\bar{A}^1_{xy}$.

Illustrative Service Table

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$	$d_x^{(i)}$	$d_x^{(r)}$
30	100,000	100	19,990	0	0
31	79,910	80	14,376	0	0
32	65,454	72	9,858	0	0
33	55,524	61	5,702	0	0
34	49,761	60	3,971	0	0
35	45,730	64	2,693	46	0
36	42,927	64	1,927	43	0
37	40,893	65	1,431	45	0
38	39,352	71	1,181	47	0
39	38,053	72	989	49	0
40	36,943	78	813	52	0
41	36,000	83	720	54	0
42	35,143	91	633	56	0
43	34,363	96	550	58	0
44	33,659	104	505	61	0
45	32,989	112	462	66	0
46	32,349	123	421	71	0
47	31,734	133	413	79	0
48	31,109	143	373	87	0
49	30,506	156	336	95	0
50	29,919	168	299	102	0
51	29,350	182	293	112	0
52	28,763	198	259	121	0
53	28,185	209	251	132	0
54	27,593	226	218	143	0
55	27,006	240	213	157	0
56	26,396	259	182	169	0
57	25,786	276	178	183	0
58	25,149	297	148	199	0
59	24,505	316	120	213	0
60	23,856	313	0	0	3,552
61	19,991	298	0	0	1,587
62	18,106	284	0	0	2,692
63	15,130	271	0	0	1,350
64	13,509	257	0	0	2,006
65	11,246	204	0	0	4,448
66	6,594	147	0	0	1,302
67	5,145	119	0	0	1,522
68	3,504	83	0	0	1,381
69	2,040	49	0	0	1,004
70	987	17	0	0	970

Interest Functions

Interest Functions at $i = 0.06$

m	$i^{(m)}$	$d^{(m)}$	$i/i^{(m)}$	$d/d^{(m)}$	$\alpha(m)$	$\beta(m)$
1	0.06000	0.05660	1.00000	1.00000	1.00000	0.00000
2	0.05913	0.05743	1.01478	0.98564	1.00021	0.25739
4	0.05870	0.05785	1.02223	0.97852	1.00027	0.38424
12	0.05841	0.05813	1.02721	0.97378	1.00028	0.46812
∞	0.05827	0.05827	1.02971	0.97142	1.00028	0.50985

where $\alpha(m) = \dfrac{id}{i^{(m)}d^{(m)}}$ and

$$\beta(m) = \frac{i - i^{(m)}}{i^{(m)}d^{(m)}}$$

Special Notes:

1. Unless specified, the force of interest is constant in each question.
2. Unless specified, future lifetimes are independent in each question.
3. Unless specified, all lives in a question follow the same mortality table.