Michigan State University
STT 456 - Actuarial Models II
Class Test 2
Wednesday, 8 April 2015
Total Marks: 100 points

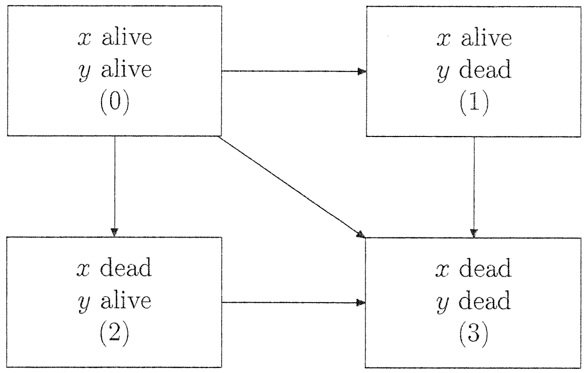
Please write your name at the space provided:

Name: Emil Valdez

- There are ten (10) multiple choice (MC) questions here and you are to answer all questions asked. Each question is worth 10 points. Partial points will be granted for those who provide detailed solutions.
- Additional question, on separate sheets, worth 20 bonus points are provided selectively to those with diligent attendance.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1: (10 points)

A four-state homogeneous Markov model represents the joint mortality of a married couple: a husband age x and a wife age y. The states are: 0 = both x and y alive; 1 = x dead, y alive; 2 = x alive, y dead, and 3 = both x and y dead.



Assume transitions occur only once at the end of each year. The one-year transition probabilities are:

$$
\begin{array}{c c}
 & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.93 & 0.04 & 0.02 & 0.01 \\ 0.00 & 0.96 & 0.00 & 0.04 \\ 0.00 & 0.00 & 0.92 & 0.08 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix}
\end{array}
$$

Calculate the probability that a married couple, both alive at the beginning of the year, will both be dead within two years.

(a) 0.0032

(b) 0.0116

(c) 0.0132

(d) 0.0148

(e) 0.0232

possible transitions	probability
$0 \to 0 \to 3$	$(.93)(.01) = .0093$
$0 \to 1 \to 3$	$(.04)(.04) = .0016$
$0 \to 2 \to 3$	$(.02)(.08) = .0016$
$0 \to 3$	0.01

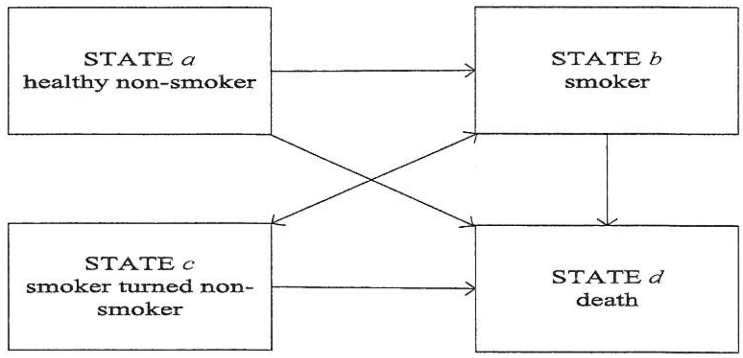
Sum = 0.0225

this choice should have been exactly 0.0225!

this is the closest!

Question No. 2: (10 points)

An insurance company determines its policy premiums according to a multiple state model, shown in the figure below, with states: (a) healthy non-smoker (who has never smoked before), (b) smoker, (c) smoker turned non-smoker, and (d) death.



The homogeneous transition probability matrix is given by:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0.7 & 0.2 & 0.0 & 0.1 \\ 0.0 & 0.5 & 0.1 & 0.4 \\ 0.0 & 0.2 & 0.5 & 0.3 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \end{array}.$$

The company issues a one-year term policy that provides death benefit of 1 at the end of the year of death. Assume transitions occur at the end of each year and that interest rate is $i = 5\%$.

Calculate the ratio of the actuarial present value of the policy for a smoker to that of a healthy non-smoker.

(a) 4

(b) 2

(c) 3/2

(d) 4/3

(e) 1

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK FOR Question No. 2

For the healthy non-smoker, transition $a \to d$

APV(death benefit) $= 0.1V$

For the smoker, transition $b \to d$

APV(death benefit) $= 0.4V$

$$\frac{\text{Smoker}}{\text{Non-Smoker}} = \frac{0.4\cancel{V}}{0.1\cancel{V}} = \underline{\underline{4.0}}$$

Question No. 3: (10 points)

A machine can be in one of four possible states, labeled a, b, c, and d. It migrates annually according to a Markov Chain with transition probabilities:

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array}$$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0.35 & 0.65 & 0.00 & 0.00 \\ 0.60 & 0.00 & 0.40 & 0.00 \\ 0.70 & 0.00 & 0.00 & 0.30 \\ 1.00 & 0.00 & 0.00 & 0.00 \end{pmatrix} \end{array} = Q$$

let this be

At time $t = 0$, the machine is in State a. A salvage company will pay 500 at the end of 2 years if the machine is in State a.

Assuming $i = 0.05$, calculate the actuarial present value at time $t = 0$ of this payment.

(a) 36.7

(b) 46.5

(c) 105.1

(d) 222.6

(e) 232.4

current state

$$\begin{array}{cccc} a & b & c & d \end{array}$$

$$(1, 0, 0, 0) * Q = (.35 \quad .65 \quad 0 \quad 0)$$

$$(.35 \quad .65 \quad 0 \quad 0) * Q = (.5125, .2275, .26, 0)$$

a

$$.35^2 + .65(.6) = .5125$$

$$.35(.65) = .2275$$

$$.65(.4) = .26$$

$$\text{APV at } t=0 = \underbrace{500 * v^2 * .5125}_{232.4263}$$

Question No. 4: (10 points)

The `Illustrative Service Table` is a multiple decrement table based on four decrements: (d) is death, (w) is withdrawal from employment, (i) is sickness, and (r) is retirement. Table is attached.

Using the table, calculate the probability that an employee, now age 62, will retire within the next 4 years.

(a) 0.36

(b) 0.42

(c) 0.50

(d) 0.58

(e) 0.64

The required probability is

$$ {}_4q_{62}^{(r)} = \frac{d_{62}^{(r)} + d_{63}^{(r)} + d_{64}^{(r)} + d_{65}^{(r)}}{l_{62}^{(\tau)}} $$

$$ = \frac{2692 + 1350 + 2006 + 4448}{18106} $$

$$ = \frac{10496}{18106} = \underline{\underline{0.5796973}} $$

Question No. 5: (10 points)

In a double decrement model with decrements 1 and 2, you are given:

- both decrements are each uniformly distributed over each year of age in the double decrement table.
- $\ell_x^{(\tau)} = 100$
- $d_x^{(1)} = 3$
- $d_x^{(2)} = 7$

Calculate $\dfrac{p_x^{\prime(2)}}{p_x^{\prime(1)}}$.

(a) 0.44

(b) 0.96

(c) 1.04

(d) 1.77

(e) 2.29

Recall: $q_x^{\prime(j)} = 1-(1-q_x^{(\tau)})^{q_x^{(j)}/q_x^{(\tau)}}$

$q_x^{(1)} = \frac{3}{100} = 0.03 \qquad q_x^{(2)} = \frac{7}{100} = 0.07$

$q_x^{(\tau)} = 0.03 + 0.07 = 0.10$

$$\frac{p_x^{\prime(2)}}{p_x^{\prime(1)}} = \frac{\left[1-q_x^{(\tau)}\right]^{q_x^{(2)}/q_x^{(\tau)}}}{\left[1-q_x^{(\tau)}\right]^{q_x^{(1)}/q_x^{(\tau)}}} = \left[1-q_x^{(\tau)}\right]^{\frac{(q_x^{(2)}-q_x^{(1)})}{q_x^{(\tau)}}}$$

$$= (1-0.10)^{0.04/0.10}$$

$$= \underline{\underline{0.9587315}}$$

Question No. 6: (10 points)

For a portfolio of fully discrete whole life insurance to (x), you are given:

- The contract annual premium is \$0.01 per dollar of death benefit.
- Renewal expenses, payable at the beginning of the year are 10% of premium plus a fixed amount of \$4.00.
- $\text{AS}_k = 1,000$ and $\text{AS}_{k+1} = 986.45$
- $\text{CV}_{k+1} = 900$
- $q^{(d)}_{x+k} = 0.02$ and $q^{(w)}_{x+k} = 0.18$
- $i = 6\%$

Calculate the amount of the death benefit.

(a) 4,000

(b) 5,000

(c) 7,500

(d) 10,000

(e) 11,000

Let DB = death benefit

$$AS_{K+1} = \frac{(AS_K + .01DB(1-.1)-4)(1.06) - .02DB - 900(.18)}{1-.02-.18}$$

Solving for DB and substituting AS, we get

$$(.02 - .9(.01)(1.06))DB = -986.45(.8) + \cancel{}\,996(1.06) - 900(.18)$$

$$0.01046\ DB = 104.6$$

$$DB = \frac{104.6}{0.01046} = \underline{\underline{10,000}}$$

Question No. 7: (10 points)

You are given:

- A husband and wife, with independent future lifetimes, are of the same age 55.
- The husband's mortality follows a constant force with $\mu^h_{55+t} = 0.04$ for all $t \geq 0$.
- The wife's mortality follows a constant force with $\mu^w_{55+t} = 0.02$ for all $t \geq 0$.
- $\delta = 5\%$

Calculate $\bar{A}_{55:55}$.

(a) 0.091

(b) 0.273

(c) 0.545

(d) 0.727

(e) 0.909

Start with annuities:

$$\bar{a}_{55:55} = \int_0^\infty v^t \, {}_tp^h_{55} \, {}_tp^w_{55} \, dt$$

$$= \int_0^\infty e^{-.05t} e^{-.04t} e^{-.02t} \, dt$$

$$= \frac{1}{.11} = \frac{100}{11}$$

$$\bar{A}_{55:55} = 1 - \delta \bar{a}_{55:55} = 1 - .05\left(\frac{100}{11}\right) = \frac{6}{11}$$

$$\approx \underline{\underline{0.545}}$$

Question No. 8:

For two lives (x) and (y) with future independent lifetimes, you are given:

- ${}_{10}p_x = 0.700$
- ${}_{10}p_y = 0.650$
- $p_{x+10} = 0.950$
- $p_{y+10} = 0.758$

Calculate the probability that both (x) and (y) will die between years 10 and 11.

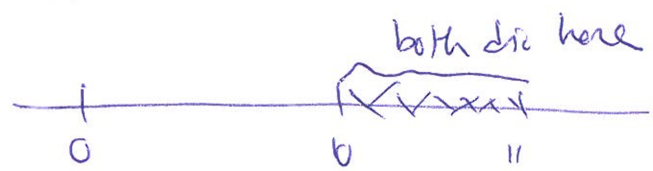
(a) 0.0055

(b) 0.0655

(c) 0.0989

(d) 0.1323

(e) 0.1923



The required probability is therefore

$${}_{10}p_x\, q_{x+10} * {}_{10}p_y\, q_{y+10}$$

$$= 0.7(1-.95) * 0.65(1-0.758)$$

$$= \underline{\underline{0.0055055}}$$

Question No. 9:

For (x) and (y) with independent future lifetimes, you are given:

- $\bar{a}_x = 11.95$
- $\bar{a}_y = 10.06$
- $\bar{a}_{\overline{xy}} = 12.59$
- $\bar{A}^{1}_{xy} = 0.25$
- $\delta = 0.06$

$\Rightarrow \bar{a}_{xy} = 11.95 + 10.06 - 12.59 = 9.42$

$\bar{A}_{xy} = 1 - \delta \bar{a}_{xy}$

$= 1 - .06(9.42) = 0.4348$

Calculate $\bar{A}_{xy}^{\;\;1}$.

(a) less than 0.07

(b) at least 0.07, but less than 0.10

(c) at least 0.10, but less than 0.13

(d) at least 0.13, but less than 0.16

(e) at least 0.16

$\bar{A}_{xy} = \bar{A}^{1}_{xy} + \bar{A}_{xy}^{\;\;1}$

so that

$\bar{A}_{xy}^{\;\;1} = \bar{A}_{xy} - \bar{A}^{1}_{xy}$

$= 0.4348 - 0.25$

$= \underline{\underline{0.1848}}$

Question No. 10:

For three lives (x), (y) and (z) with independent future lifetimes, which of the following gives the correct integral for ${}_nq_{\overset{2}{x}y\overset{1}{z}}$?

(a) $\displaystyle\int_0^n {}_tp_y\, {}_tq_{\overline{xz}}\, \mu_{x+t}dt$

(b) $\displaystyle\int_0^n {}_tp_{xy}\, {}_tq_z\, \mu_{x+t}dt$ ✓

(c) $\displaystyle\int_0^n {}_tq_{xz}\, {}_tp_y\, \mu_{y+t}dt$

(d) $\displaystyle\int_0^n {}_tp_{xyz}\, \mu_{z+t}dt$

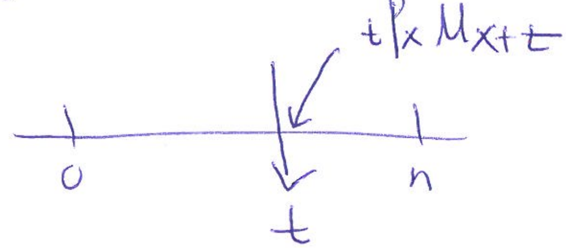
(e) $\displaystyle\int_0^n {}_tp_{yz}\, {}_tq_x\, \mu_{z+t}dt$

Z dies first

X next

y last

From the point of view of (X), when he/she dies,
(z) must have died but (y) still alive



$$\int_0^n {}_tp_y\, {}_tq_z\, {}_tp_x\, \mu_{x+t}\, dt$$

$$= \int_0^n {}_tq_z\, {}_tp_{xy}\, \mu_{x+t}\, dt$$

The other choices are not correct!

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK