Michigan State University

STT 456 - Actuarial Models II

Class Test 1

Friday, 27 February 2015

Total Marks: 100 points

Please write your name at the space provided:

Name: EMIL VALDEZ

- There are ten (10) multiple choice (MC) questions here and you are to answer all questions asked. Each question is worth 10 points. Partial points will be granted for those who provide detailed solutions.
- Additional question, on separate sheets, worth 20 bonus points are provided selectively to those with diligent attendance.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1: (10 points) d @ 2δ is ${}^2d = 1 - v^2 = 1 - 1/1.055^2$

ABC Insurance Company sells N fully discrete whole life insurance policies with death benefit of $100, each with the same age x. You are given:

- All policies have independent future lifetimes.
- $i = 5.5\%$ $\Rightarrow d = .055/1.055$ and $v = 1/1.055$
- $\ddot{a}_x = 9.95$ $\Rightarrow A_x = 1 - d\ddot{a}_x = 1 - \frac{.055}{1.055}(9.95) = 0.4812796$
- ${}^2\ddot{a}_x = 7.14$ (This is life annuity calculated at 2δ.) ${}^2A_x = 1 - {}^2d\,{}^2\ddot{a}_x = 0.2749502$
- The annual contract premium is $5.33 per policy.
- The 99th percentile on a standard normal distribution is 2.326.

Determine the smallest N so that ABC has at least a 99% probability of a gain from this portfolio of policies.

If P = premium per \$1 of insurance, $P = \frac{5.33}{100} = 0.0533$.

(a) 1

(b) 118

(c) 399 ✓

(d) 1,232

(e) 3,715

$$E[L_{0,i}] = B\left(1+\frac{P}{d}\right)A_x - P/d = -4.905538$$

$$Var[L_{0,i}] = B^2(1+P/d)^2\left[{}^2A_x - (A_x)^2\right] = 1771.823$$

$$L_{agg} = \sum_{i=1}^{N} L_{0,i} \Rightarrow E[L_{agg}] = -4.905538N$$

and $Var[L_{agg}] = 1771.823N$

$$Pr(\text{gain}) = Pr(L_{agg} < 0) \approx Pr\Big(Z < \underbrace{\frac{-4.905538N}{\sqrt{1771.823N}}}_{\leq -2.326}\Big) \geq 0.99$$

$$\Leftrightarrow \frac{-4.905538}{\sqrt{177.823}}\frac{\cancel{N}}{\cancel{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\sqrt{\cancel{N}}} \leq -2.326$$

$$\Leftrightarrow (\sqrt{N}) \geq \left(\frac{2.326\sqrt{1771.823}}{4.905538}\right)^2 = 398.3516$$

$$\Leftrightarrow N \geq 398.3516 \quad \text{Choose } N = 399$$

Question No. 2: (10 points) Let B= death benefit , $v = 1/1.05$

For a fully discrete whole life insurance issued to (x), you are given:

- $i = 5\%$
- $q_{x+10} = 0.0268$
- $q_{x+11} = 0.0288$
- ${}_{10.5}V = 343.29$
- ${}_{10.7}V = 340.48$
- Deaths are uniformly distributed over each year of age. $\Rightarrow$ ${}_{0.5}q_{x+10} = 0.5\, q_{x+10}$
  ${}_{0.7}q_{x+10} = 0.7\, q_{x+10}$

$343.29(1-{}_{.5}q_{x+10}) = (V_{10}+P)(1.05)^{.5} - B\,{}_{0.5}q_{x+10}\,v^{.5}$

$340.48(1-{}_{.7}q_{x+10}) = (V_{10}+P)(1.05)^{.7} - B\,{}_{0.7}q_{x+10}\,v^{.3}$

Calculate the amount of the death benefit (to the nearest hundred).

(a) 1,000

(b) 1,100

(c) 1,300

(d) 1,400

✓ (e) 1,500

Rewriting the two equations to each solve first for $V_{10}+P$,

$V_{10}+P = 343.29(1-.5q_{x+10})(1.05)^{.5} + B\,0.5q_{x+10}$

$V_{10}+P = 340.48(1-.7q_{x+10})(1.05)^{.3} + B\,0.7q_{x+10}$

Take ratio and solve for B, we get

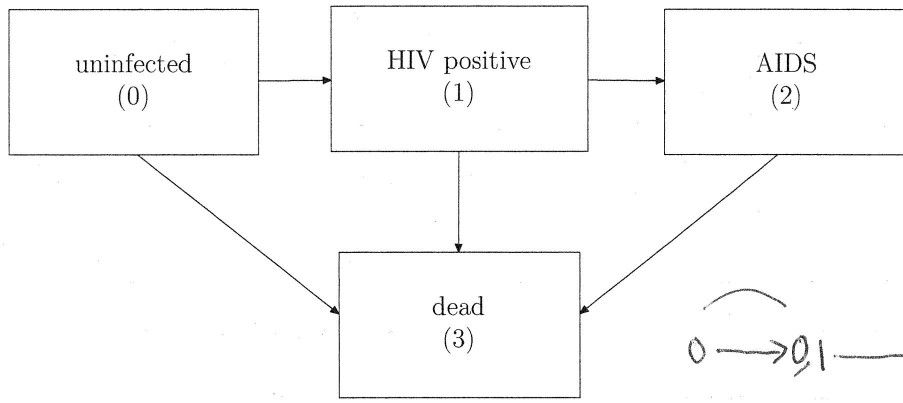
$$B\,\cancel{0.2q_{x+10}} = \frac{343.29(1-.5q_{x+10})(1.05)^{.5} - 340.48(1-.7q_{x+10})(1.05)^{.3}}{0.2q_{x+10}}$$

Plug $q_{x+10} = 0.0268$,

$B = \underline{1499.104}$

Question No. 3: (10 points)

You are given the following multiple state model:



$0 \longrightarrow 0,1 \longrightarrow 1,3$

$0 \quad s \quad t \quad 10$

The forces of transition are the following:

- $\mu^{01} = 0.004$
- $\mu^{03} = 0.006$
- $\mu^{12} = 0.400$
- $\mu^{13} = 0.200$
- $\mu^{23} = 0.350$

$$\int_0^{10}\int_0^t .004\, e^{-.006s} \cdot 0.200\, e^{-.6(t-s)}\, ds\, dt$$

$$(.004)(.200)\int_0^{10}\int_0^t e^{-.600t}\, e^{.594s}\, ds\, dt$$

Calculate the probability that an uninfected person will be infected with HIV, that never developed to AIDS, and die within 10 years.

(a) less than 0.01

(b) at least 0.01, but less than 0.02

(c) at least 0.02, but less than 0.03

(d) at least 0.03, but less than 0.04

(e) at least 0.04

$$\frac{(.004)(.200)}{.594}\int_0^{10} e^{-.6t}\left(e^{.594t}-1\right) dt$$

$$\int_0^{10}\left(e^{-.006t}-e^{-.6t}\right) dt$$

$$\underbrace{\frac{(.004)(.200)}{.594}\cdot\left[\frac{1}{.006}\left(1-e^{-.06}\right)-\frac{1}{.6}\left(1-e^{-6}\right)\right]}$$

0.01083283

Question No. 4: (10 points)

For a fully discrete whole life policy of $100,000 issued to (50), you are given:

- Expenses, incurred at the beginning of each year, are summarized below:

	% of Premium	Per Policy
First year	25%	15
Renewal years	5%	5

- Gross premium is determined according to the actuarial equivalence principle.
- $i = 4\%$ $\quad d = .04/1.04$
- $\ddot{a}_{50} = 11.007$ $\quad \Rightarrow A_{50} = 1 - \frac{.04}{1.04}(11.007) = 0.5766538$
- $\ddot{a}_{55} = 9.450$ $\quad \Rightarrow A_{55} = 1 - \frac{.04}{1.04}(9.450) = 0.6365385$

Calculate ${}_5V^g$, the fifth-year gross premium reserve (to the nearest ten).

(a) 12,190

(b) 13,170

(c) 14,150

(d) 15,130

(e) 16,110

First calculate G: $APVFG_0 = APVFB_0 + APVFE_0$

$$G\ddot{a}_{50} = 100000 A_{50} + .20G + 10 + (.05G+5)\ddot{a}_{50}$$

$$\cancel{(.95\ddot{a}_{50} - .20)}G = \frac{100000 A_{50} + 10 + 5\ddot{a}_{50}}{.95\ddot{a}_{50} - .20}$$

$$G = 5628.358$$

$${}_5V^g = APVFB_5 + APVFE_5 - APVFG_5$$

$$= 100000 A_{55} + (.05G + 5)\ddot{a}_{55} - G\ddot{a}_{55}$$

$$= 100000(0.6365385) + (5 - .95G)(9.450)$$

$$= \underline{\underline{13,170.05}}$$

Question No. 5: (10 points)

For a whole life insurance policy of \$1 issued to (x), you are given:

- Death benefit is payable at the end of the year of death.
- Level premiums are payable annually at the beginning of each year, only for the first n years. There are no premiums after n years.
- Reserves are calculated based on the Full Preliminary Term (FPT) method.

Determine β in the FPT calculation.

(a) $\dfrac{A_{x+1}}{\ddot{a}_{x:\overline{n}|}}$

(b) $\dfrac{A_x - vq_x}{a_{x:\overline{n}|}}$

(c) $\dfrac{A_{x+1:\overline{n-1}|}}{\ddot{a}_{x+1:\overline{n-1}|}}$

✓ (d) $\dfrac{A_{x+1}}{\ddot{a}_{x+1:\overline{n-1}|}}$

(e) $\dfrac{A_{x+1}}{\ddot{a}_{x:\overline{n}|} - 1}$

$P = A_x / \ddot{a}_{x:\overline{n}|}$

Replace P's with α, and renewal β

$$\underbrace{P\ddot{a}_{x:\overline{n}|}}_{A_x} = \underset{\downarrow\ vq_x}{\alpha} + \beta(\ddot{a}_{x:\overline{n}|} - 1)$$

$$\frac{A_x - vq_x}{\ddot{a}_{x:\overline{n}|} - 1} = \beta$$

OR

$$\beta = \frac{\cancel{vp_x}\, A_{x+1}}{\cancel{vp_x}\, \ddot{a}_{x+1:\overline{n-1}|}} = \frac{A_{x+1}}{\ddot{a}_{x+1:\overline{n-1}|}}$$

Question No. 6: (10 points)

For a 10-year endowment insurance on (55), you are given:

- The death benefit, payable at the end of the year of death, is equal to 2500 plus the benefit reserve.
- The endowment benefit is 5000.
- Level premiums, P, are payable annually at the beginning of each year.
- $q_{55+k} = 0.01$, for $k = 0, 1, 2, \ldots$
- $i = 5\%$

use recursive formula

Calculate P.

$_0V = 0$

(a) 355

$$_1V = \frac{P(1.05) - (2500 + \cancel{_1V})(0.01)}{\cancel{1 - .01}} = P(1.05) - 2500(.01)$$

(b) 367

(c) 375

$$_2V(1-\cancel{.01}) = (P(1.05) - 2500(.01) + P)(1.05) - (2500 + \cancel{_2V})(.01)$$

(d) 390

$$_2V = P\left[(1.05)^2 + 1.05\right] - \underbrace{2500(.01)}_{25}\left[1.05 + 1\right]$$

✓ (e) 402

⋮

$$_{10}V = P \cdot \sum_{t=1}^{10} (1.05)^t - 25 \sum_{t=0}^{9} 1.05^t$$

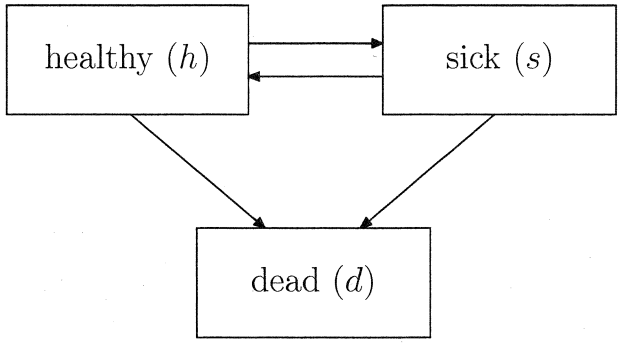
5000, the endowment

Solving for P, we get

$$P = \frac{5000 + 25\,\frac{1.05^{10} - 1}{.05}}{\frac{1.05^{10} - 1}{.05}\,1.05} = \underline{\underline{402.4027}}$$

Question No. 7: (10 points)

You are given the following health-sickness multiple state model:



$1 - .99740 - .00097$
$= .00163$

In calculating transition probabilities, the Euler method is being used with a step size of $h = \frac{1}{4}$. Transition intensities and some estimated probabilities for the first year are given below:

t	μ^{hs}_{50+t}	μ^{hd}_{50+t}	μ^{sh}_{50+t}	μ^{sd}_{50+t}	${}_tp^{hh}_{50}$	${}_tp^{hs}_{50}$	${}_tp^{hd}_{50}$	
0.00	0.00387	0.00653	0.00077	0.00653	1.00000	0.00000	0.00000	
0.25	0.00399	0.00666	0.00080	0.00666	0.99740	0.00097	–	.00163
0.50	0.00412	0.00680	0.00082	0.00680	0.99475	0.00196	–	.00329
0.75	0.00425	0.00693	0.00085	0.00693	0.99203	0.00298	–	.00499
1.00	0.00438	0.00708	0.00088	0.00708	–	–	–	

Give the estimate for ${}_1p^{hd}_{50}$.

(a) less than 0.0058

(b) at least 0.0058, but less than 0.0062

✓ (c) at least 0.0062, but less than 0.0068

(d) at least 0.0068, but less than 0.0072

(e) at least 0.0072

Writing the KFE for ${}_tp^{hd}_{50}$:

$$\frac{d}{dt}\,{}_tp^{hd}_{50} = {}_tp^{hs}_{50}\,\mu^{sd}_{50+t} + {}_tp^{hh}_{50}\,\mu^{hd}_{50+t}$$

Using Euler's with $h = 0.25$:

$${}_1p^{hd}_{50} \approx {}_{.75}p^{hd}_{50} + 0.25\left[{}_{.75}p^{hs}_{50}\,\mu^{sd}_{50.75} + {}_{.75}p^{hh}_{50}\,\mu^{hd}_{50.75}\right]$$

$$.00499 + 0.25\left[.00298(.00693) + .99203(.00693)\right]$$

$$= 0.00671385\underline{5}$$

Question No. 8:

$G = 2600$

An insurer issued 400,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2005. Each policy issued has a death benefit of $100,000 with an annual gross premium of $2,600.

You are given:

1/1/2005 — 1/1/2014 — 1/1/2015

0 — 9 — 10

this period we are interested in

- The following values in Year 2014:

	anticipated	actual
Expenses as a percent of premium	0.05	0.06
Annual effective rate of interest	0.02	0.05
q_{59}	0.0085	0.0090

- The gross premium reserves per policy at the end of Year 2013 and Year 2014, respectively, are:

$$_{9}V = 2,044.32 \quad \text{and} \quad _{10}V = 2,324.13$$

- A total of 385,100 remain in force at the beginning of Year 2014.

- Gains and losses are calculated in the following order: interest then expenses then mortality.

Calculate the total gain (or loss) ~~due to interest~~ for this portfolio of policies in Year 2014.

(a) a loss of 23.7 million

(b) a loss of 22.8 million

(c) zero gain or loss

✓ (d) a gain of 22.8 million

(e) a gain of 23.7 million

$$\text{Actual} = \underbrace{385100 * ({}_9V + 0.94G)(1.05) - 385100 * (100{,}000 - {}_{10}V)(.0090)}_{1{,}476{,}339{,}836}$$

$$\text{Expected} = \underbrace{385100 * ({}_9V + 0.95G)(1.02) - 385100 * (100{,}000 - {}_{10}V)(0.0085)}_{1{,}453{,}506{,}616}$$

$$\text{Actual} - \text{Expected} = +22{,}833{,}220$$

Despite losses in expenses and mortality, there is more than enough offset coming from gain due to interest!

Question No. 9:

For a fully discrete 10-year deferred whole life insurance of \$1 on (45), you are given:

- The annual benefit premium is payable only during the deferred period and no premiums are to be paid after the deferred period.
- $A_{55} = 0.3895$;
- ${}_{10}E_{45} = 0.6588$;
- $\ddot{a}_{45} = 18.8230$;
- $q_{54} = 0.004$; and
- $i = 4\%$.

Calculate the ${}_9V^n$, the net premium reserve at the end of 9 years.

(a) less than 0.30

✓(b) at least 0.30, but less than 0.35

(c) at least 0.35, but less than 0.40

(d) at least 0.40, but less than 0.45

(e) at least 0.45

First calculate premium

$$P = \frac{{}_{10}E_{45} A_{55}}{\ddot{a}_{45:\overline{10}|}} = \frac{.6588(.3895)}{8.3659} = 0.0307$$

$$\frac{1-A_{55}}{d}$$

where $\ddot{a}_{45:\overline{10}|} = \ddot{a}_{45} - {}_{10}E_{45}\, \ddot{a}_{55}$

$$= 18.8230 - .06588\left(\frac{1-.3895}{.04/1.04}\right)$$

$$= 8.3659$$

Then ${}_{10}V = A_{55} = 0.3895$ since no more premiums after deferred period.

next use recursive formula

$$({}_9V + P)(1+i) = {}_{10}V + (1 - {}_{10}V)\, q_{54}$$

$$({}_9V + .0307)(1.04) = \underbrace{.3895 + (1-.3895)(.004)}_{.391942}$$

Solving for ${}_9V$, we get

$$_9V = \frac{.391942}{1.04} - .0307 = \underline{\underline{0.3462}}$$

Question No. 10:

For a fully discrete whole life insurance of $10,000 issued to (50), you are given:

- Mortality follows the `Illlustrative Life Table`.
- $i = 6\%$.

Calculate ${}_{15}V^n$, the net premium reserve at the end of 15 years.

(a) less than 1000

(b) at least 1000, but less than 1500

(c) at least 1500, but less than 2000

(d) at least 2000, but less than 2500

✓ (e) at least 2500

$$P = 10{,}000 \frac{A_{50}}{\ddot{a}_{50}} = 10{,}000 \frac{0.24905}{13.2668} = 187.7242$$

$$\begin{aligned} {}_{15}V^n &= APV(FB_{15}) - APV(FP_{15}) \\ &= 10{,}000 A_{65} - P\,\ddot{a}_{65} \\ &= \underbrace{10{,}000(0.43980) - 187.7242(9.8969)} \\ &= 2540.112 \end{aligned}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK