Exercise 6.9

Let G the required gross monthly premium.

The APV of the 20-year deferred annuity benefits with an initial annual payment of 50,000 increasing by 2% thereafter is given by

$$\begin{aligned}\text{APV(benefits)} &= \sum_{20}^{\infty} 50000(1.02)^{k-20} v^k \, {}_kp_{[40]} \\ &= \frac{50000}{1.02^{20}} \left[\sum_{20}^{\infty} (1.02v)^k \, {}_kp_{[40]} \right] \\ &= \frac{50000}{1.02^{20}} \left({}_{20|}\ddot{a}_{[40]} \right)_{i_1},\end{aligned}$$

where $\left({}_{20|}\ddot{a}_{[40]} \right)_{i_1}$ is a 20-year deferred annuity evaluated at interest rate $i_1 = (1.05/1.02) - 1$. It can be verified that based on the Standard Select Survival Model, we have

$$\left({}_{20|}\ddot{a}_{[40]} \right)_{i_1} = 10.18434.$$

The APV of the expenses can be found using

$$\begin{aligned}\text{APV(expenses)} &= 0.025(50000) + 0.15G + 0.05(12G)\, \ddot{a}^{(12)}_{[40]:\overline{20|}} + \sum_{0}^{\infty} 20(1.03)^{k+1} v^{k+1} \, {}_kp_{[40]} \, q_{[40]+k} \\ &= 1250 + 0.15G + 0.60G\, \ddot{a}^{(12)}_{[40]:\overline{20|}} + 20\left(A_{[40]}\right)_{i_2},\end{aligned}$$

where $\left(A_{[40]}\right)_{i_2}$ is a whole life insurance of 1 with benefit payable at the end of the year of death, evaluated at interest rate $i_2 = (1.05/1.03) - 1$. It can be verified that based on the Standard Select Survival Model, we have

$$\left(A_{[40]}\right)_{i_2} = 0.4245105.$$

The APV of the monthly gross premiums is given by

$$\text{APV(premiums)} = 12G\, \ddot{a}^{(12)}_{[40]:\overline{20|}},$$

where we can approximate the temporary annuity using the Woolhouse formula, with three terms:

$$\ddot{a}^{(12)}_{[40]:\overline{20|}} \approx \ddot{a}_{[40]:\overline{20|}} - \frac{11}{24}\left(1 - {}_{20}E_{[40]}\right) - \frac{12^2 - 1}{12(12^2)} \left[\delta + \mu_{[40]} - {}_{20}E_{[40]}\left(\delta + \mu_{60}\right)\right],$$

where $\delta = \log(1.05)$ and

$$\begin{aligned}{}_{20}E_{[40]} &= v^{20} \frac{\ell_{60}}{\ell_{[40]}} = (1.05)^{-20} \frac{96634.14}{99327.82} = 0.3666686 \\ \ddot{a}_{[40]:\overline{20|}} &= \ddot{a}_{[40]} - {}_{20}E_{[40]}\, \ddot{a}_{60} = 18.45956 - 0.3666686(14.90407) = 12.99471 \\ \mu_{[40]} &= (0.9)^2(A + Bc^{40}) = 0.0004128936 \\ \mu_{60} &= A + Bc^{60} = 0.003221528\end{aligned}$$

Plug these values, we get

$$\ddot{a}^{(12)}_{[40]:\overline{20}|} \approx 12.70194.$$

Equating APV(benefits) + APV(expenses) with APV(premiums), we solve the monthly gross premium with

$$\begin{aligned} G &= \frac{(50000/1.02^{20})\left({}_{20|}\ddot{a}_{[40]}\right)_{i_1} + 1250 + 20\left(A_{[40]}\right)_{i_2}}{11.4\,\ddot{a}^{(12)}_{[40]:\overline{20}|} - 0.15} \\ &= \frac{(50000/1.02^{20})(10.18434) + 1250 + 20(0.4245105)}{11.4(12.70194) - 0.15} \\ &= 2377.754. \end{aligned}$$