Exercise 6.7

(a) Let P be the net single premium. The net future loss random variable can be written as

$$L_0 = P\,v^{K+1} I(K < 19) + 40000 v^{20}\, \ddot{a}_{\overline{K+1-20}|} - P$$

(b) Solving for P, we get

$$P = P\,A^{\;1}_{[45]:\overline{20}|} + 40000\,{}_{20}E_{[45]}\,\ddot{a}_{65}$$

so that

$$\begin{aligned} P &= \frac{40000\,{}_{20}E_{[45]}\,\ddot{a}_{65}}{1 - A^{\;1}_{[45]:\overline{20}|}} \\ &= \frac{40000(0.35999)(13.550)}{1-(0.15149-0.35999(0.35477))} = \frac{195114.6}{0.9762237} = 199,866.7 \end{aligned}$$

(c) With the 5 year guarantee, the actuarial present value of benefits can be expressed as

$$\text{APV(benefits)} = P\,A^{\;1}_{[45]:\overline{20}|} + 40000\,{}_{20}E_{[45]}\left(\ddot{a}_{\overline{5}|} + {}_{5}E_{65}\,\ddot{a}_{70}\right)$$

so that

$$\begin{aligned} P &= \frac{40000\,{}_{20}E_{[45]}\left(\ddot{a}_{\overline{5}|} + {}_{5}E_{65}\,\ddot{a}_{70}\right)}{1 - A^{\;1}_{[45]:\overline{20}|}} \\ &= \frac{40000(0.35999)\left(\dfrac{1-v^5}{d} + 0.75455(12.008)\right)}{1-(0.15149-0.35999(0.35477))} \\ &= \frac{195929.4}{0.9762237} = 200,701.4 \end{aligned}$$