Exercise 6.3

(a) Let B be the amount of death benefit, payable at the end of year of death and $K = K_{[41]}$, the curtate future lifetime of select age 41. Then the loss-at-issue random variable can be expressed as

$$L_0 = Bv^{K+1} - 350\,\ddot{a}_{\overline{K+1|}}, \quad \text{for } K = 0, 1, 2$$

and $L_0 = -350\,\ddot{a}_{\overline{2|}}$ for $K \geq 3$. Details for calculating B are summarized in the table below:

k	(1)	(2)	(3)	(1) × (2)	350 × (1) × (3)
0	0.00113224	0.943396	1.00000	0.00107	0.39628
1	0.00187371	0.889996	1.94340	0.00167	1.27448
2	0.00219434	0.839619	2.83339	0.00184	2.17610
≥ 3	0.99479970	0.000000	2.83339	0.00000	986.53036
sum	1.00000			0.004578162	990.3772

where

$$\begin{aligned}
(1) &= \Pr[K = k] \text{ for } k = 0, 1, 2 \text{ and } \Pr[K \geq 3] \text{ otherwise} \\
(2) &= v^{k+1} \text{ for } k = 0, 1, 2 \text{ and } 0 \text{ otherwise} \\
(3) &= \ddot{a}_{\overline{k+1|}} \text{ for } k = 0, 1, 2 \text{ and } \ddot{a}_{\overline{2|}} \text{ otherwise}
\end{aligned}$$

The probabilities are computed based on

$$\begin{aligned}
\Pr[K = 0] &= \frac{d_{[41]}}{\ell_{[41]}} = \frac{113}{99802} \\
\Pr[K = 1] &= \frac{d_{[41]+1}}{\ell_{[41]}} = \frac{187}{99802} \\
\Pr[K = 2] &= \frac{d_{[41]+2}}{\ell_{[41]}} = \frac{219}{99802} \\
\Pr[K \geq 3] &= 1 - \frac{113 + 187 + 219}{99802}
\end{aligned}$$

By the equivalence principle with $\mathrm{E}[L_0] = 0$, we have

$$B = \frac{990.3772}{0.004578162} = 216,326.4.$$

One can also verify that you can compute the death benefit based on

$$B = \frac{350\,A^{\;1}_{[41]:\overline{3|}}}{\ddot{a}_{[41]:\overline{3|}}} = \frac{(350)(2.829649)}{0.004578162} = 216,326.4.$$

(b) The loss-at-issue random variable now can be expressed as

$$L_0 = 216326.4v^{K+1} - 350\,\ddot{a}_{\overline{K+1|}} \quad \text{for } K = 0, 1, 2$$

and $L_0 = -350\,\ddot{a}_{\overline{2|}}$ for $K \geq 3$. Details for calculating standard deviation of L_0 are summarized in the table below:

k	$\Pr[K = k]$	loss	loss $\times \Pr[K = k]$	loss$^2 \times \Pr[K = k]$
0	0.00113224	203731.4910	230.6733	46995419.0
1	0.00187371	191849.5198	359.4703	68964214.7
2	0.00219434	180640.1130	396.3867	71603337.1
≥ 3	0.99479970	-991.6874	-986.5304	978329.8
sum	1.00000		0.0000	188541300.0

We indeed do not need the calculation of $\mathrm{E}[L_0]$ because according to the equivalence principle, this is zero, as also confirmed in the table above. Hence, the standard deviation of L_0 is

$$\mathrm{SD}[L_0] = \sqrt{\mathrm{Var}[L_0]} = \sqrt{188541300.0} = 13,731.03.$$

(c) Note from the table that the loss-at-issue is positive for $k = 0, 1, 2$; always a positive loss if death is prior to the end of the term of the contract. Thus, the required probability is

$$\Pr[L_0 > 0] = \Pr[K \leq 2] = \frac{(113 + 187 + 219)}{99802} = 0.005200297.$$