Exercise 6.14

Let G be the monthly gross premium.
The APV of the death benefits is

$$\text{APV(benefits)} = 100000\, A^{(12)}_{\overset{1}{[50]}:\overline{10}|}$$

The APV of the expenses is

$$\text{APV(expenses)} = 100 + 0.15G + 0.15(12G)\, \ddot{a}^{(12)}_{[50]:\overline{1}|} + 0.05(12G)\, \ddot{a}^{(12)}_{[50]:\overline{10}|} + 250\, A^{(12)}_{\overset{1}{[50]}:\overline{10}|}$$

The APV of the premiums is

$$\text{APV(premiums)} = 12G\, \ddot{a}^{(12)}_{[50]:\overline{10}|}$$

Equating APV(benefits) + APV(expenses) with APV(premiums), we solve the monthly gross premium with

$$G = \frac{100250\, A^{(12)}_{\overset{1}{[50]}:\overline{10}|} + 100}{11.4\, \ddot{a}^{(12)}_{[50]:\overline{10}|} - 1.8\, \ddot{a}^{(12)}_{[50]:\overline{1}|}}.$$

where, based on the claims acceleration approach, we have

$$\begin{aligned}
A^{(12)}_{\overset{1}{[50]}:\overline{10}|} &= (1+i)^{11/24}\big(A_{[50]} - {}_{10}E_{[50]}\, A_{60}\big) \\
&= (1+i)^{11/24}\left[\big(1-(1-v)\ddot{a}_{[50]}\big) - v^{10}\frac{\ell_{60}}{\ell_{[50]}}\big(1-(1-v)\ddot{a}_{60}\big)\right] \\
&= (1.05)^{11/24}\Big\{[1-(1-(1.05)^{-1})(17.02835)] \\
&\qquad -(1.05)^{-10}\frac{96634.14}{98552.51}[1-(1-(1.05)^{-1})(14.90407)]\Big\} \\
&= 0.01471224.
\end{aligned}$$

We approximate the temporary life annuities using the Woolhouse formula, with three terms.

$$\ddot{a}^{(12)}_{[50]:\overline{10}|} \approx \ddot{a}_{[50]:\overline{10}|} - \frac{11}{24}\big(1 - {}_{10}E_{[50]}\big) - \frac{12^2-1}{12(12^2)}\left[\delta + \mu_{[50]} - {}_{10}E_{[50]}\,(\delta + \mu_{60})\right],$$

where $\delta = \log(1.05)$ and

$$\begin{aligned}
{}_{10}E_{[50]} &= v^{10}\frac{\ell_{60}}{\ell_{[50]}} = (1.05)^{-10}\frac{96634.14}{98552.51} = 0.6019631 \\
\ddot{a}_{[50]:\overline{10}|} &= \ddot{a}_{[50]} - {}_{10}E_{[50]}\,\ddot{a}_{60} = 17.02835 - 0.6019631(14.90407) = 8.056649 \\
\mu_{[50]} &= (0.9)^2(A + Bc^{50}) = 0.000933578 \\
\mu_{60} &= A + Bc^{60} = 0.003221528
\end{aligned}$$

Plug these values, we get

$$\ddot{a}^{(12)}_{[50]:\overline{10}|} \approx 7.87692.$$

Similarly, we have

$$\ddot{a}^{(12)}_{[50]:\overline{1}|} \approx \ddot{a}_{[50]:\overline{1}|} - \frac{11}{24}\left(1 - {}_1E_{[50]}\right) - \frac{12^2-1}{12(12^2)}\left[\delta + \mu_{[50]} - {}_1E_{[50]}\left(\delta + \mu_{[50]+1}\right)\right],$$

where

$$\begin{aligned} {}_1E_{[50]} &= v\,\frac{\ell_{[50]+1}}{\ell_{[50]}} = (1.05)^{-1}\,\frac{98450.67}{98552.51} = 0.9513968 \\ \ddot{a}_{[50]:\overline{1}|} &= \ddot{a}_{[50]} - {}_1E_{[50]}\,\ddot{a}_{[50]+1} = 17.02835 - 0.9513968(16.84718) = 0.9999968 \\ \mu_{[50]+1} &= (0.9)(A + Bc^{51}) = 0.001141383 \end{aligned}$$

so that

$$\ddot{a}^{(12)}_{[50]:\overline{1}|} \approx 0.9775367.$$

The monthly gross premium is therefore

$$G = \frac{100250(0.01471224) + 100}{11.4(7.87692) - 1.8(0.9999968)} = 17.89883.$$

I believe the answer in the book gives the annual gross premium.