Exercise 6.10

Let G the required gross annual premium.
With premiums payable only up to 10 years, we have the APV of gross premiums given by

$$\text{APV(premiums)} = G\,\ddot{a}_{[40]:\overline{10}|},$$

where, based on the Standard Select Survival Model,

$$\begin{aligned}\ddot{a}_{[40]:\overline{10}|} &= \ddot{a}_{[40]} - {}_{10}E_{[40]}\,\ddot{a}_{50} \\ &= 18.45956 - (1.05)^{-10}\,\frac{98576.37}{99327.82}(17.02453) = 8.087045.\end{aligned}$$

The APV of the 20-year term insurance policy is

$$\text{APV(benefits)} = 100000\,A^{\,1}_{[40]:\overline{20}|},$$

where (see Exercise 6-9),

$$\begin{aligned}A^{\,1}_{[40]:\overline{20}|} &= A_{[40]:\overline{20}|} - {}_{20}E_{[40]} \\ &= 1 - (1-v)\ddot{a}_{[40]:\overline{20}|} - {}_{20}E_{[40]} \\ &= 1 - (1-(1/1.05))(12.99471) - 0.3666686 = 0.01453590.\end{aligned}$$

The APV of the expenses can be found using

$$\begin{aligned}\text{APV(expenses)} &= 0.04G\ddot{a}_{[40]:\overline{10}|} + 0.20G + 0.05G\ddot{a}_{[40]:\overline{10}|} + 5\ddot{a}_{[40]:\overline{20}|} + 5 \\ &= 0.09G\ddot{a}_{[40]:\overline{10}|} + 0.20G + 5\ddot{a}_{[40]:\overline{20}|} + 5.\end{aligned}$$

Equating APV(benefits) + APV(expenses) with APV(premiums), the gross annual premium then is calculated as

$$\begin{aligned}G &= \frac{100000\,A^{\,1}_{[40]:\overline{20}|} + 5\ddot{a}_{[40]:\overline{20}|} + 5}{0.91\,\ddot{a}_{[40]:\overline{10}|} - 0.20} \\ &= \frac{100000(0.01453590) + 5(12.99471) + 5}{0.91(8.087045) - 0.20} \\ &= 212.8117.\end{aligned}$$