Exercise 6.1

(a) Let P be the required annual benefit premium. The loss-at-issue random variable can be written as

$$L_0 = \text{PVFB}_0 - \text{PVFP}_0 = 200000v^{K+1} - P\ddot{a}_{\overline{\min(K+1,20)}|}$$

(b) According to the equivalence principle, we set $\text{E}[L_0] = 0$ to solve for P:

$$P = 200000 \times \frac{A_{[30]}}{\ddot{a}_{[30]:\overline{20}|}},$$

where

$$\begin{aligned} A_{[30]} &= 0.07693 \\ \ddot{a}_{[30]:\overline{20}|} &= \ddot{a}_{[30]} - {}_{20}E_{[30]}\,\ddot{a}_{50} = 19.384 - 0.37256(17.025) = 13.04117 \end{aligned}$$

Therefore, we have

$$P = 200000 \times \frac{0.07693}{13.04117} = 1179.802.$$

(c) The event $L_0 < 0$ is equivalent to the event

$$200000v^{K+1} - P\ddot{a}_{\overline{\min(K+1,20)}|} < 0.$$

When $K = 19$, we can verify that $L_0 = 59939.80$ so that $K > 19$. Therefore, this event is equivalent to

$$200000v^{K+1} - P\ddot{a}_{\overline{20}|} < 0.$$

or equivalently

$$K > (-1/\delta)\log\left(\frac{P\ddot{a}_{\overline{20}|}}{200000}\right) - 1 = 51.49991.$$

Finally, we have

$$\begin{aligned} \Pr[L_0 < 0] &= \Pr[K > 51.49991] = \Pr[K \geq 52] \\ &= {}_{52}p_{[30]} = \frac{\ell_{82}}{\ell_{[30]}} = \frac{70507.19}{99721.06} = 0.7070441. \end{aligned}$$

The contract makes a profit only if the person select age 30 will survive another 52 years. The answer in the book is different and it appears that it might be computing the probability of surviving 53 years, and this will be incorrect.